Day/Date 4th May, 18 / FRIDAY

-: LECTURE # 7 :-

REVISION

PROBLEM # 1 :-

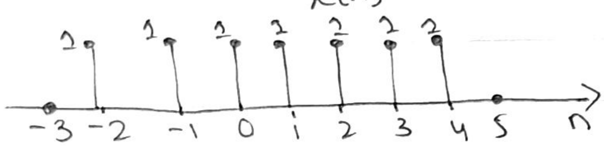
Let $x[n]$ be a signal with $x[n] = 0$ for $n < -2$ and $n > 4$. For each signal given below, determine the values of n for which it is guaranteed to be zero;

a) $x[-n+2]$

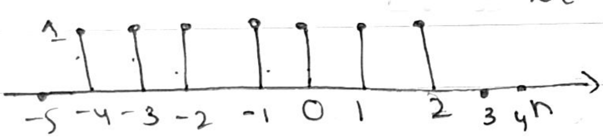
Sol:-

The signal $x[n]$ is flipped and flipped signal is shifted by 2 to the right. This new signal will be zero for $n < -2$ and $n > 4$.

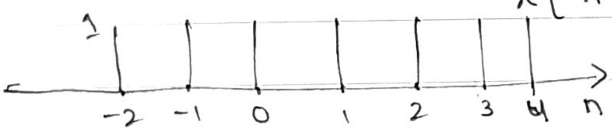
$x[n]$



$x[n+2]$



$x[-n+2]$



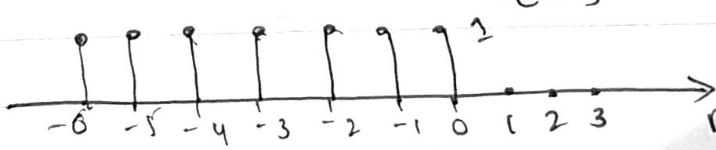
$n < -2$ and $n > 4$ are all 0.

b) $x[n+4]$

Sol:

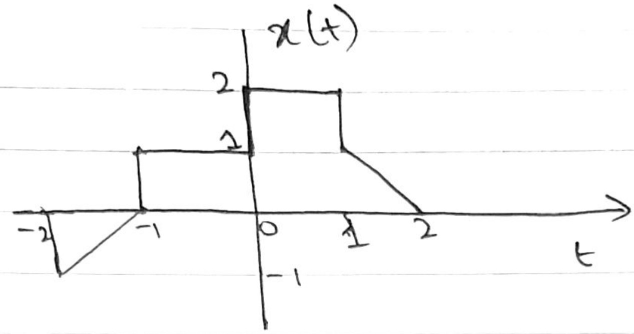
The signal $x[n]$ is shifted by 4 to the left. The shifted signal will be zero for $n < -6$ and $n > 0$.

$x[n+4]$



PROBLEM #2:

A continuous-time signal $x(t)$ is shown in figure below. Sketch and label carefully each of the following signals:

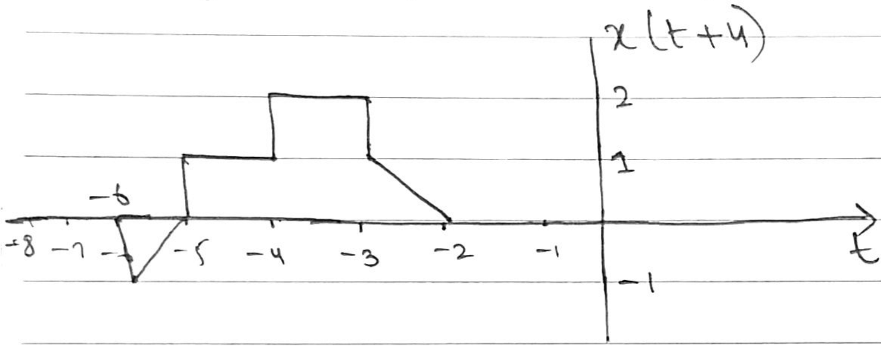


a) $x(4-t/2)$

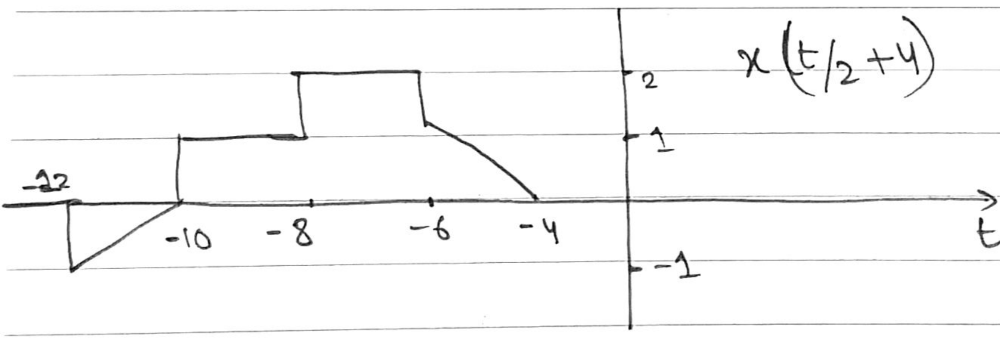
Sol:

$x(4-t/2)$

Shift the signal $x(t) \Rightarrow 4$ points left.

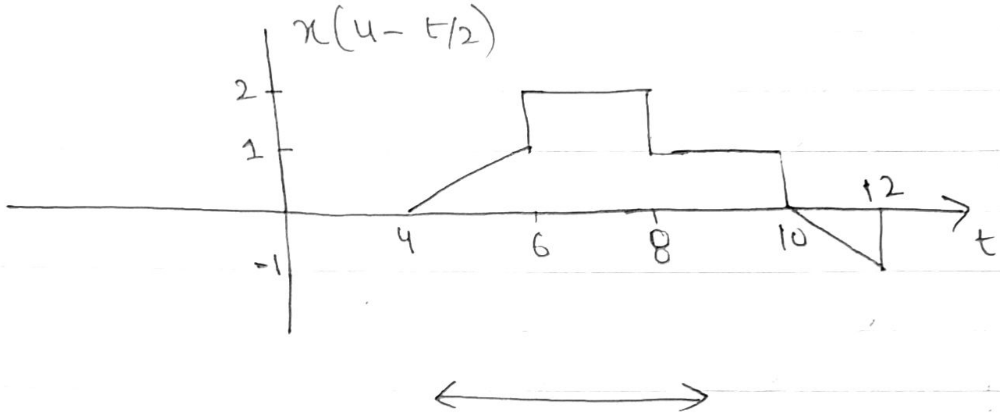


Now scale $x(t+4) \rightarrow \frac{1}{2}$



Day/Date

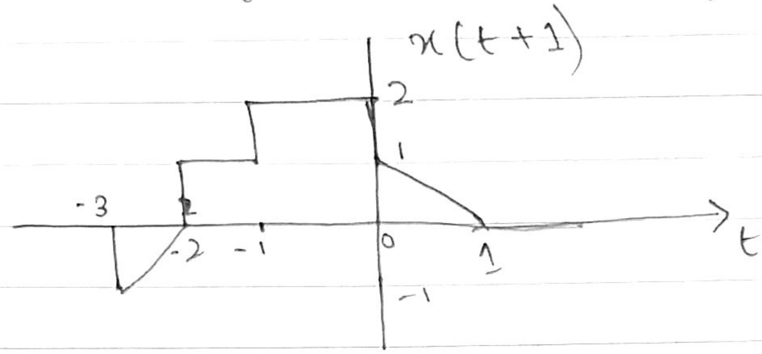
Now flip $x(t/2 + 4)$



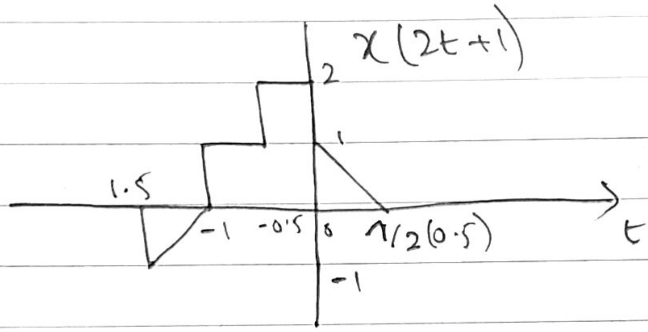
b) $x(2t+1)$

Sol:-

Let's shift $x(t)$ to 1 point left.

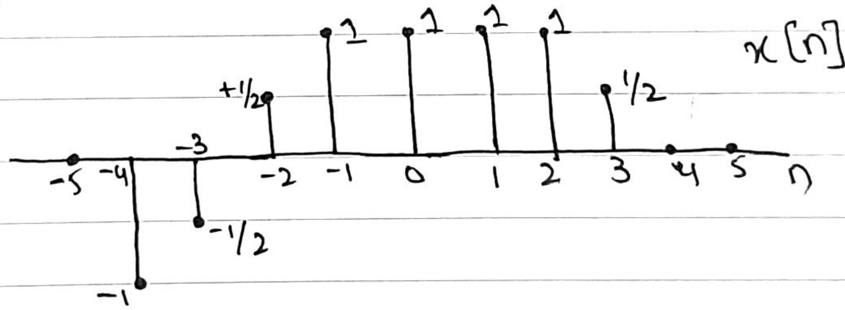


Now scale $x(t+1)$ with 2



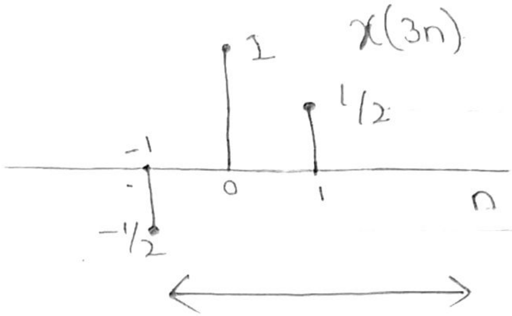
PROBLEM #3:

A discrete-time signal $x(t)$ is shown in figure below. Sketch and label carefully each of the following signals:

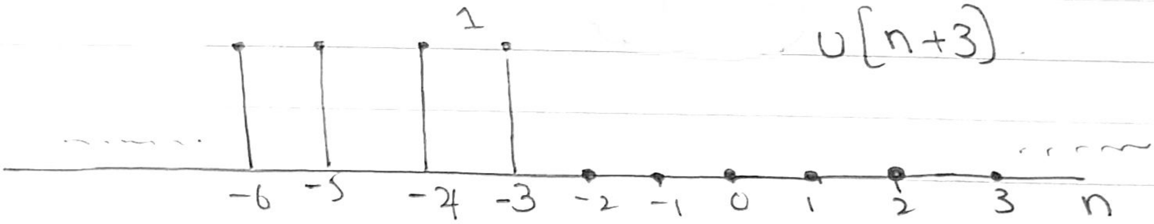


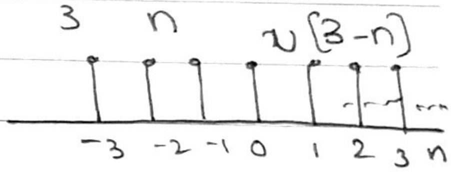
a) $x(3n)$

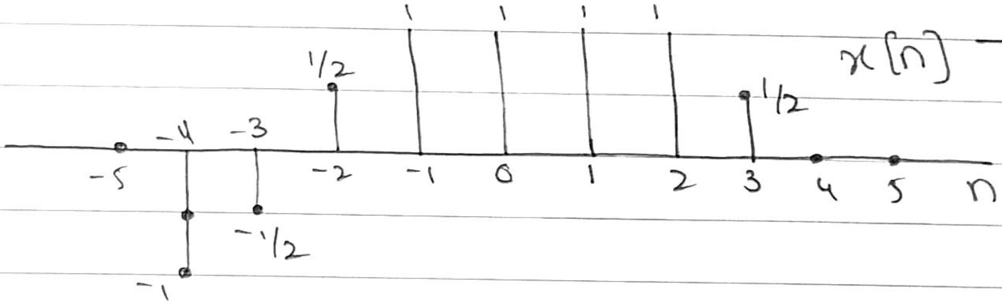
Solo



b) $x[n] \cdot u[3-n]$

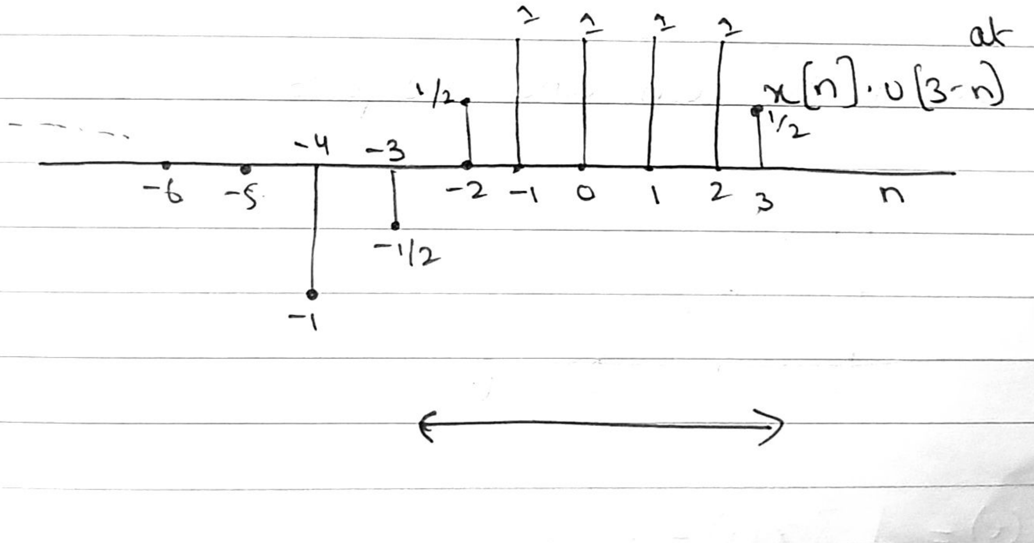






at $-4 \Rightarrow (-1)\cdot(1) \Rightarrow -1$

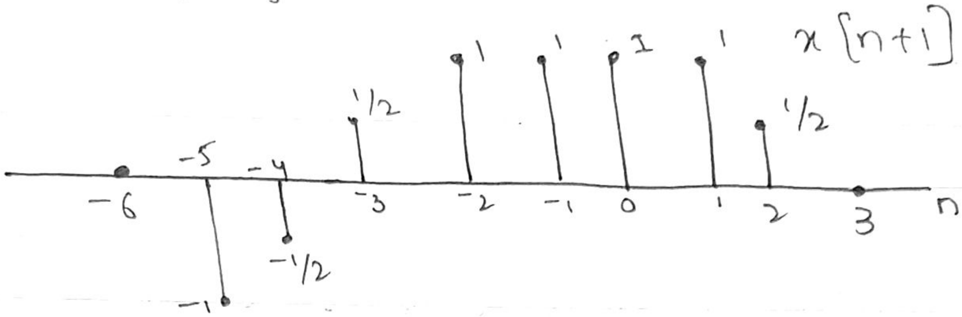
at $-3 \Rightarrow (1)\cdot(-1/2) \Rightarrow -1/2$



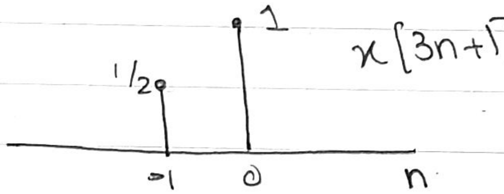
c) $x[3n+1]$

Sol:

Advance $x[n+1]$



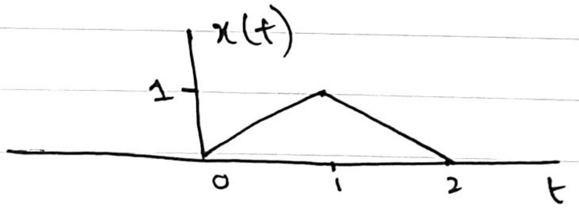
Now scale $x[n+1]$ by 3



PROBLEM # 4:-

Determine and sketch the even and odd parts of the signals shown below :-

a)

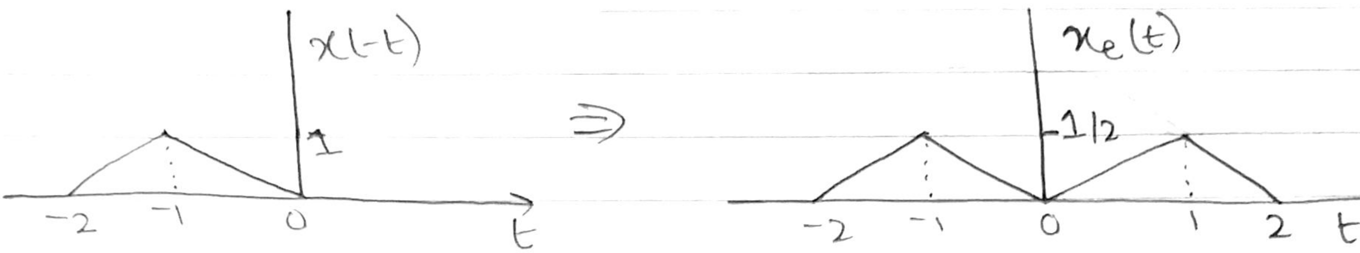


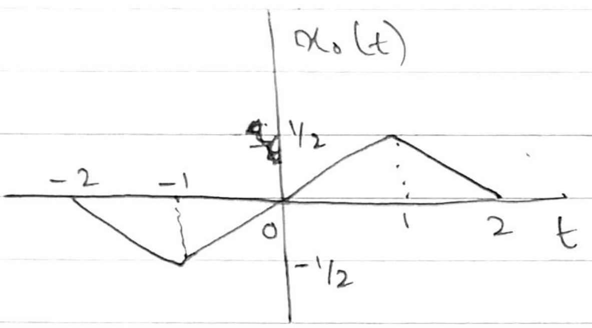
Sol:

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

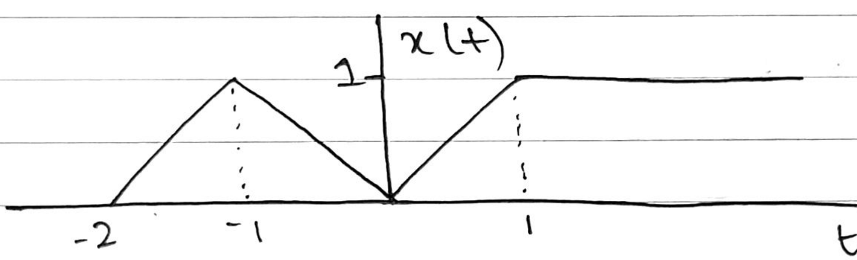
$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

$x(-t)$

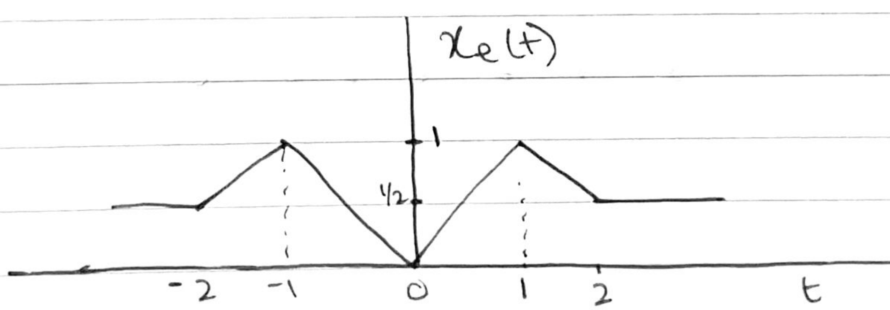

x(-t)
1
-2
-1
0
t
$\Rightarrow$
$x_e(t)$
1/2
-2
-1
0
1
2
t

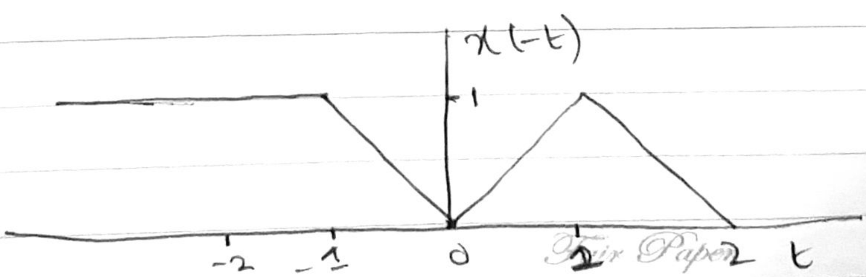

$x_o(t)$
1/2
-2
-1
0
1
2
t
-1/2

b)

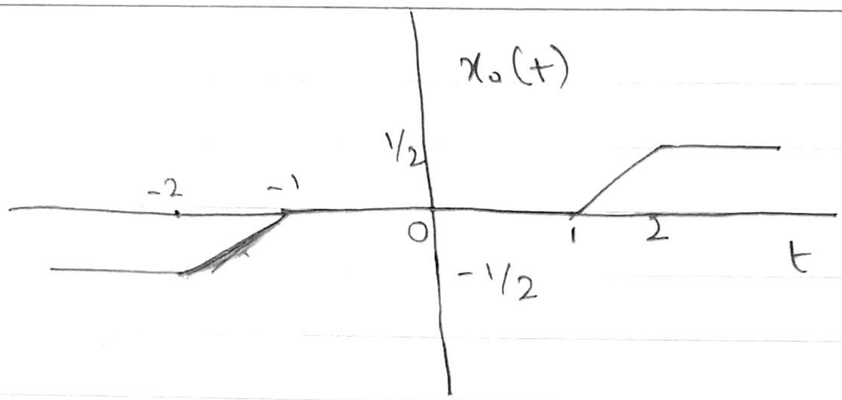

x(t)
1
-2
-1
1
t

Sol:-


$x_e(t)$
1
1/2
-2
-1
0
1
2
t

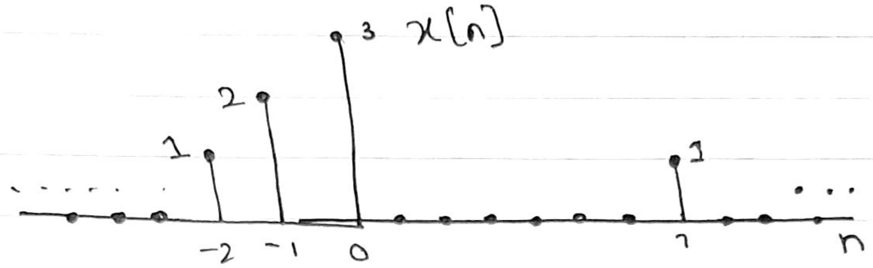

x(-t)
1
-2
-1
0
1
2
t

Day/Date

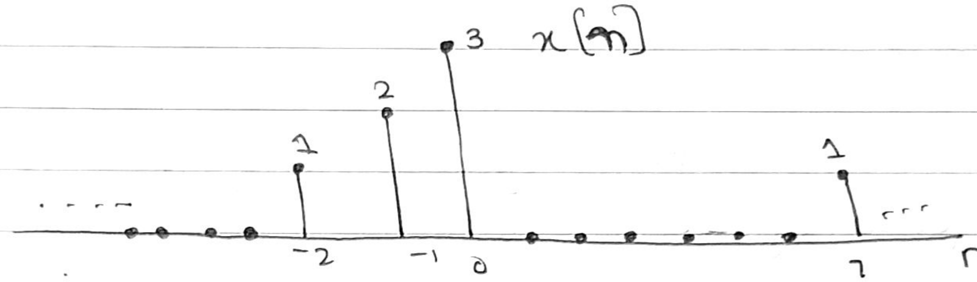


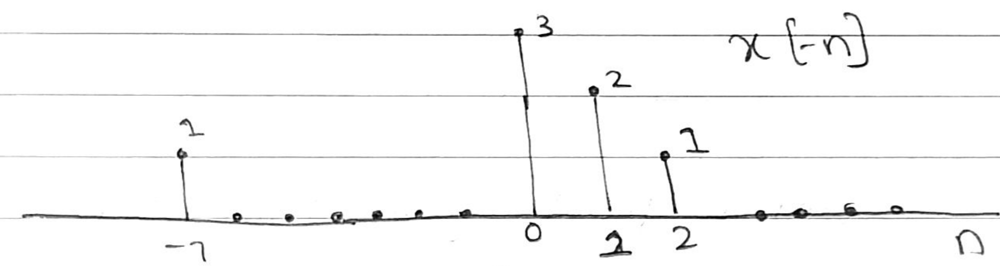
PROBLEM # 5:

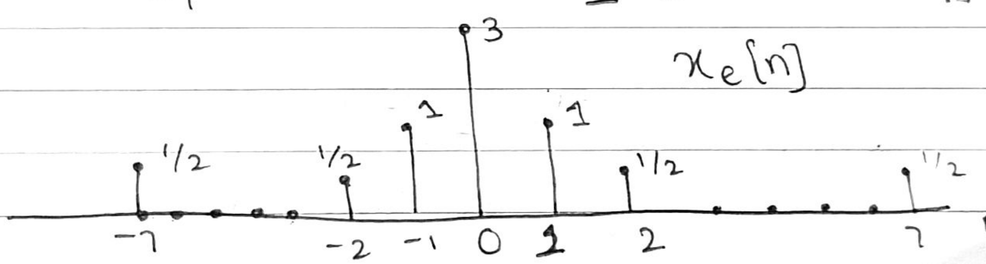
Determine and sketch the even and odd parts of the signals shown below:

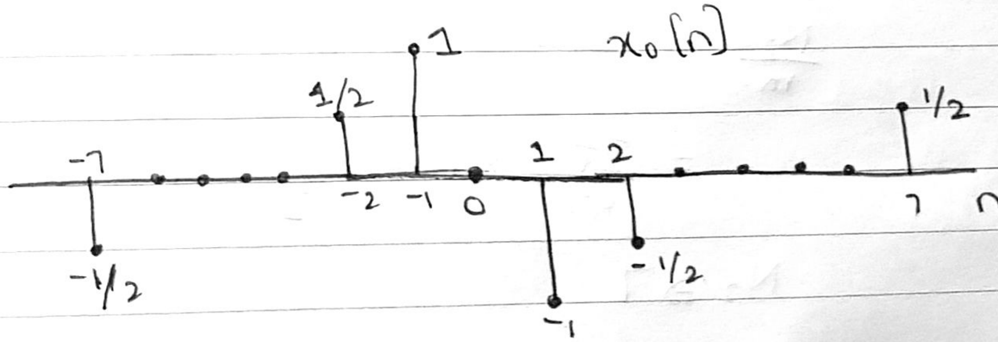


Sol:









PROBLEM # 6:-

Determine whether or not each of the following signals is periodic?

a) $x_1(t) = 2e^{i(t+\pi/4)}\,u(t)$

Sol:-

This given signal is not periodic as it is 0 for $t < 0$ due to unit step $u(t)$ function.

b) $x_2[n] = u[n] + u[-n]$

Sol:-

$x_2[n] = 1$ for all n. Therefore, it is periodic with a fundamental period of 1.

PROBLEM # 7:-

Determine the fundamental period of the signal $x(t) = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$.

Sol:-

$$x(t) = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$$

Step: 1 Calculate periods for all the functions seperately.

$N_1 = 1$

$$\frac{N_2}{m} = \frac{2\pi}{w_0} \qquad \therefore w_0 = \frac{4\pi}{7}$$

$$= \frac{\cancel{2\pi}}{\cancel{4\pi}} \times 7 \Rightarrow \frac{7}{2} \quad \text{where } m = 2$$

$N_2 \Rightarrow 7$

Day/Date

$$\frac{N_3}{m} = \frac{2\pi}{\omega_0} \qquad \therefore \omega_0 = \frac{2\pi}{5}$$

$$\frac{N_3}{m} = \frac{2\pi}{2\pi} \times 5 \Rightarrow 5 \qquad \therefore m = 1$$

$$N_3 = 5.$$

Step: 2 Check whether the given signal is periodic or not

$$\frac{N_1}{N_2}, \frac{N_1}{N_3}$$

$\frac{1}{7}, \frac{1}{5}$ as both are rational then the given signal is periodic.

Step: 3 Calculate N_0.

N_0 (LCM of N_1, N_2, N_3) $= 35$

7	7, 5, 1
5	1, 5, 1
1	1, 1, 1

The fundamental period for $x[n]$ is 35.

Day/Date

PROBLEM # 8:-

$$S_1: \; y_1[n] = 2x_1[n] + 4x_1[n-1]$$
$$S_2: \; y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3]$$

The system is in series.

a) Determine the input-output relationship for system S.

Sol:-

$$x_1[n] \longrightarrow \boxed{S_1} \xrightarrow{y_1[n]} \boxed{S_2} \xrightarrow{y_2[n]}$$

The signal $x_2[n]$, which is the input to S_2, is the same as $y_1[n]$.
Therefore,

$$y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3]$$
$$= y_1[n-2] + \frac{1}{2}y_1[n-3]$$
$$= 2x_1[n-2] + 4x_1[n-1-2] + \frac{1}{2}\left[2x_1[n-3] + 4x_1[n-1-3]\right]$$
$$= 2x_1[n-2] + 4x_1[n-3] + \frac{\cancel{2}^{1}}{\cancel{2}}x_1[n-3] + \frac{\cancel{4}^{2}}{\cancel{2}_{1}}x_1[n-4]$$

$$y_2[n] = 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4]$$

The input-output relationship for S is
$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

$\longleftrightarrow$

b) Does the input-output relationship ~~for~~ of a system S change if the order in which S_1 and S_2 are connected in series is reversed (i.e., if S_2 follows S_1)?

Sol:-
The input-output relationship does not change if the order in which S_1 and S_2 are connected in series is reversed. We can easily prove this by assuming that S_1 follows S_2. In this case, the signal $x_1[n]$, which is the input to S_1 is the same as $y_2[n]$. Therefore,

$$y_1[n] = 2x_1[n] + 4x_1[n-1]$$
$$= 2y_2[n] + 4y_2[n-1]$$
$$= 2\left(x_2[n-2] + \frac{1}{2}x_2[n-3]\right) + 4\left[x_2(n-3) + \frac{1}{2}x_2[n-4]\right)$$
$$= 2x_2[n-2] + \frac{\cancel{2}^{1}}{\cancel{2}}x_2[n-3] + 4x_2[n-3] + \frac{\cancel{4}^{2}}{\cancel{2}_{1}}x_2[n-4]$$

$$y_1[n] \Rightarrow 2x_2[n-2] + 5x_2[n-3] + 2x_2[n-4]$$

The input-output relation for S is once again
$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

PROBLEM # 9:
Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by: $y(t) = x(\sin(t))$

a) Is this system causal?

Sol:
$$y(t) = x(\sin(t))$$
The system is not causal because the output $y(t)$ at some time may depend on future values of $x(t)$. For example $y(-\pi) = x(0)$.

b) Is this system linear?

Sol:
Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

Day/Date

$$x_1(t) \longrightarrow y_1(t) = x_1(\sin(t))$$
$$x_2(t) \longrightarrow y_2(t) = x_2(\sin(t))$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$.
That is:

$$x_3(t) = ax_1(t) + bx_2(t)$$

Where a and b are arbitrary scalars, If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t)$ is,

$$\begin{aligned} y_3(t) &= x_3(\sin(t)) \\ &= a x_1(\sin(t)) + b x_2(\sin(t)) \\ &= ay_1(t) + by_2(t). \end{aligned}$$

Therefore, the system is linear.

PROBLEM #10 :-

$$y[n] = x[n-2] - 2x[n-8]$$

1) Memoryless :-

Sol:

The system is clearly not memoryless as it depends on ~~past~~ past values of the input. Hence, it is with memory system.

2) Time Invariant :-

Sol:

ST-1:

$$y(n) \xrightarrow{n_0} y(n-n_0) = x[n-2-n_0] - 2x[n-n_0-8] = y_1[n]$$

ST-2:

$$x[n] \xrightarrow{n_0} x(n-n_0) \Rightarrow sys \rightarrow x[(n-n_0)-2] - 2x[(n-n_0)-8]$$
$$= x[n-n_0-2] - 2x[n-n_0-8] = y_2[n]$$

$y_1[n] = y_2[n]$. Hence, the system is time invariant.

Day/Date

3) Linear :-

Sol:-

a) law of addition:-

$x_1[n] \rightarrow sys \rightarrow y_1[n] = x_1[n-2] - 2x_1[n-8]$

$x_2[n] \rightarrow sys \rightarrow y_2[n] = x_2[n-2] - 2x_2[n-8]$

$y_1[n] + y_2[n] = x_1[n-2] - 2x_1[n-8] + x_2[n-2] - 2x_2[n-8] \Rightarrow y_1(n)$

$x_1[n] + x_2[n] \rightarrow sys \rightarrow y'(n) = x_1[n-2] - 2x_1[n-8] + x_2[n-2]$
$- 2x_2[n-8]$

$y_1[n] = y'[n]$ law satisfied.

a) law of homogenity:-

S-1:- $x[n] \rightarrow sys \rightarrow y[n] \rightarrow 'k' \rightarrow ky[n] \Rightarrow k x[n-2] - 2kx[n-8]$

S-2:- $x[n] \rightarrow 'k' \rightarrow kx[n] \rightarrow sys \rightarrow y'[n] \Rightarrow k x[n-2] - 2kx[n-8]$

as both are equal so law is satisfied.

Since of law of addition and law of homogenity is satisfied, hence the system is lineal.

4) Causal:-

Sol:-

Since the given output is dependent on past values of i/p, hence the system is Causal.

5) Stable:-

Sol:-

Since for the given function the i/p amplitude is between $-\infty$ to ∞ the system is stable.

Day/Date

PROBLEM # 11:-

The following are the impulse responses of LTI systems. Determine whether each system is causal and /or stable. Justify your answer:

a) $h[n] = \left(\frac{1}{2}\right)^n u[-n]$

Sol:-

Anti-causal because $h[n] = 0$ for $n > 0$.

Unstable because $\sum_{n=-\infty}^{0} (1/2)^n = \infty$.

$\longleftrightarrow$

b) $h(t) = e^{-4t} u(t-2)$

Sol:-

Causal because $h(t) = 0$ for $t < 0$.

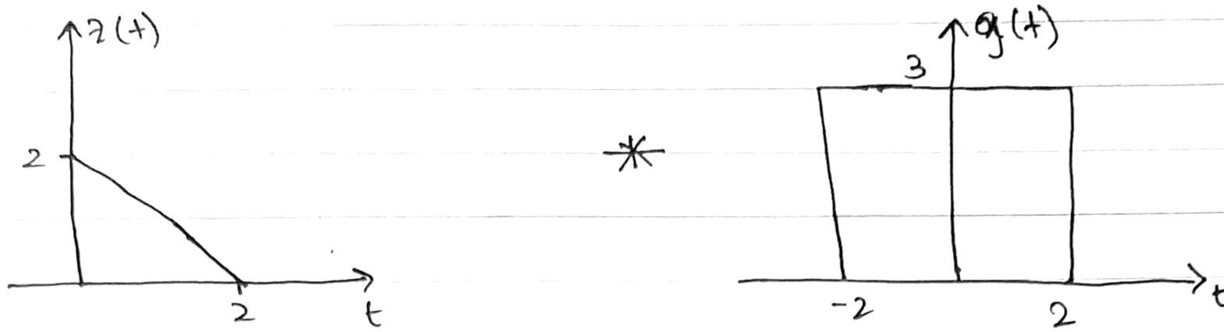
Stable because $\int_{-\infty}^{\infty} |h(t)| dt = \int_{2}^{\infty} e^{-4t} dt$

$= -\frac{e^{-4t}}{4}\Big|_2^{\infty} = \left[\frac{-e^{-\infty}}{4} - \left(\frac{-e^{-4(2)}}{4}\right)\right]$

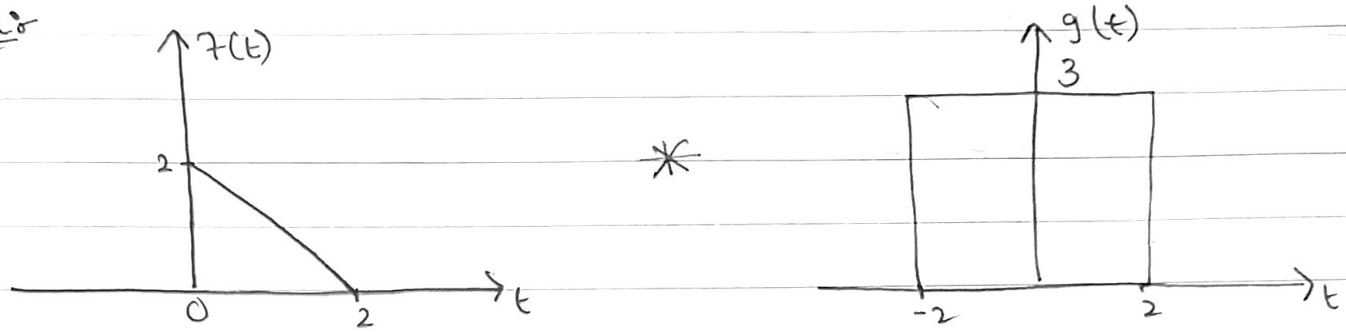
$= 0 + \frac{e^{-8}}{4} \Rightarrow \frac{e^{-8}}{4} < \infty.$

PROBLEM #12:-

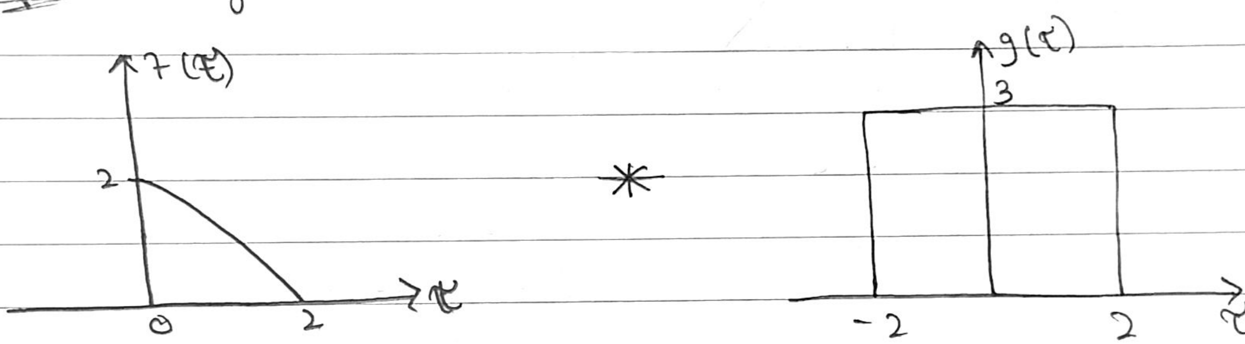
Convolve the following two functions:-



Sol:-

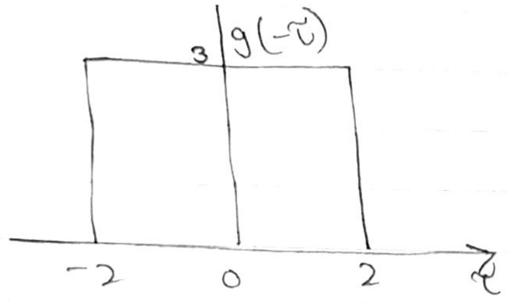


Step:1 Change $t \to \tau$

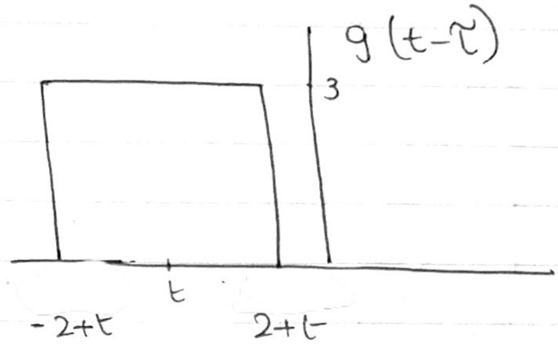


Day/Date

Step :2 Flip $g(\tau)$ and shift

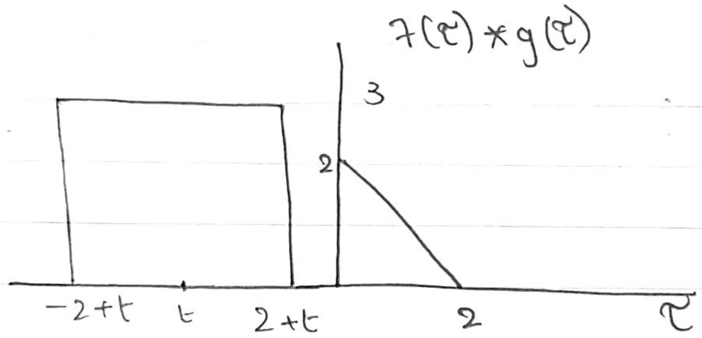


$\Rightarrow$



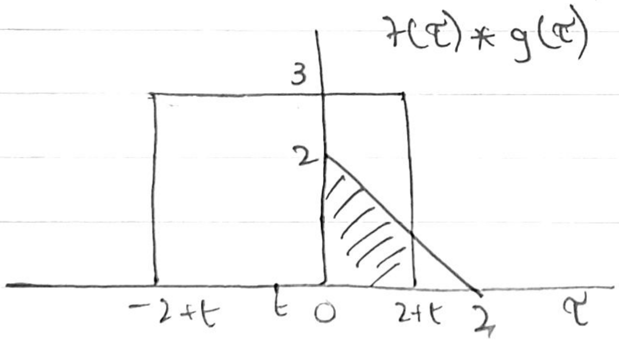
Step :3 Now integrate $f(\tau) * g(\tau)$

Interval #1 :- $t < -2$



$y(t) = 0$ as there is no overlapping

Interval #2 :- $-2 \leq t < 0$



$$y(t) = \int_0^{2+t} 3(-\tau+2)\, d\tau$$

$$= \int_0^{2+t} (-3\tau + 6)\, d\tau$$

$$= -3\int_0^{2+t} \tau\, d\tau + 6\int_0^{2+t} d\tau$$

$$= -3\left|\frac{\tau^2}{2}\right|_0^{2+t} + 6\left|\tau\right|_0^{2+t}$$

$$= -3\left[\frac{(2+t)^2}{2} - 0\right] + 6\left[2+t-0\right]$$

Day/Date

$y(t) = -3\left[\frac{4+2t+t^2}{2}\right] + 6[2+t]$

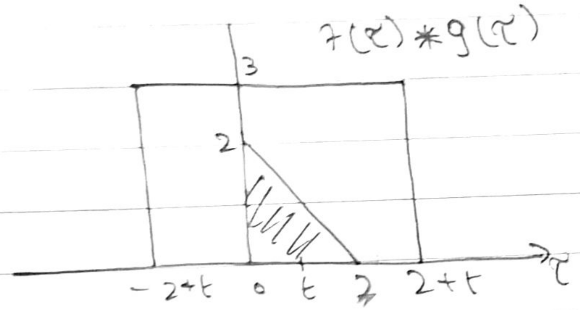
$= \frac{-12-12t-3t^2}{2} + 12 + 6t$

$= \frac{-12 \cancel{-12t} - 3t^2 + 24 + \cancel{12t}}{2}$

$y(t) = \frac{-3t^2+12}{2} = \frac{-3t^2}{2} + \frac{\cancel{12}^{6}}{\cancel{2}_{1}}$

$y(t) \Rightarrow \frac{-3t^2}{2} + 6$

Interval 3:- $0 \le t < 2$



$y(t) = \int_0^2 3(-\tau+2)\,d\tau$

$= -3\int_0^2 \tau\,d\tau + 6\int_0^2 d\tau$

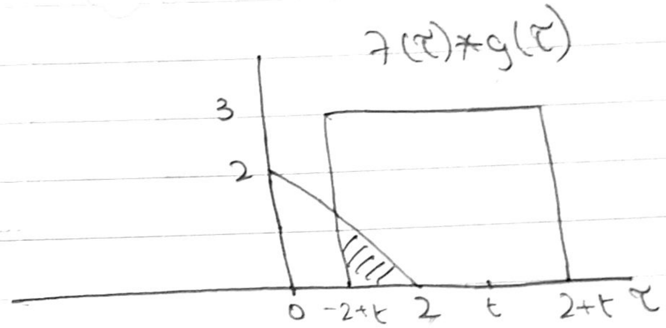
$= -3\left[\frac{\tau^2}{2}\right]_0^2 + 6|\tau|_0^2$

$= -3\left[\frac{2^2}{2} - 0\right] + 6[2-0]$

$= -3\left[\frac{\cancel{4}^{2}}{\cancel{2}_{1}}\right] + 6[2] = -6 + 12$

$y(t) \Rightarrow 6$

Interval 4:- $2 \le t < 4$



$y(t) = \int_{-2+t}^{2} 3(-\tau+2)\,d\tau$

$= -3\int_{-2+t}^{2} \tau\,d\tau + 6\int_{-2+t}^{2} d\tau$

$= -3\left[\frac{\tau^2}{2}\right]_{-2+t}^{2} + 6\left[\tau\right]_{-2+t}^{2}$

$= -3\left[\frac{\cancel{4}^{2}}{\cancel{2}_{1}} - \frac{(-2+t)^2}{2}\right] + 6[2-(-2+t)]$

$= -3\left[2 - \frac{(t^2-4t+4)}{2}\right] + 6[2+2-t]$

$= -3\left[\frac{\cancel{4} - t^2 + 4t \cancel{-4}}{2}\right] + 6[4-t]$

$= -3\left[\frac{4t-t^2}{2}\right] + 24 - 6t$

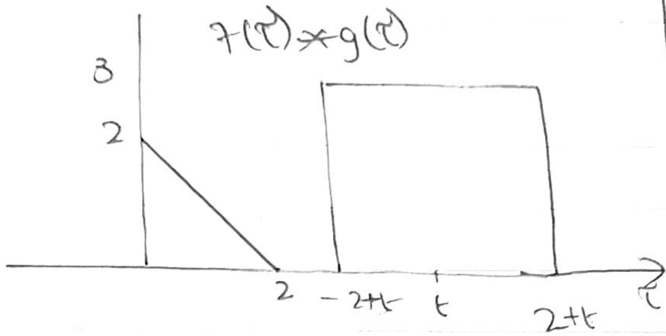
$= \frac{-\cancel{12}^{6}t}{\cancel{2}_{1}} + \frac{3t^2}{2} + 24 - 6t$

Day/Date

$$y(t) = \frac{3t^2}{2} + 24 - 6t - 6t$$

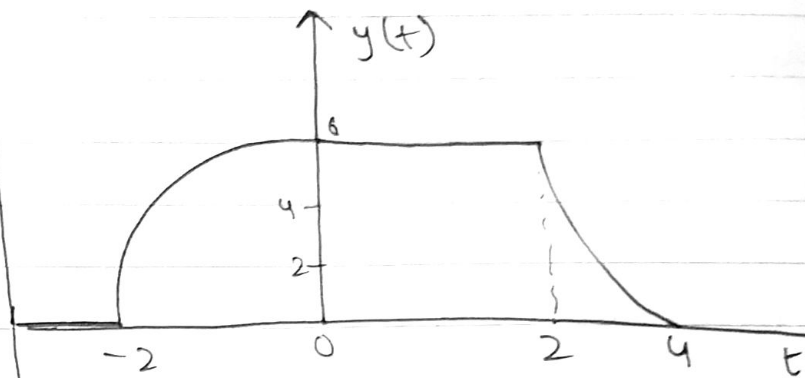
$$y(t) = \frac{3t^2}{2} + 24 - 12t$$

Interval 5: $t \geq 4$



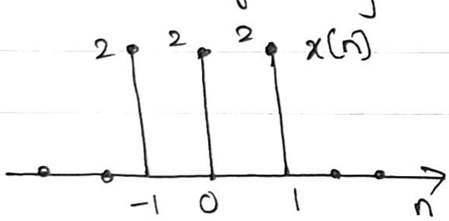
No overlapping so $y(t) = 0$

$$y(t) = \begin{cases} 0 & , t < -2 \\ -\frac{3t^2}{2} + 6 & , -2 \leq t < 0 \\ 6 & , 0 \leq t < 2 \\ \frac{3t^2}{2} - 12t + 24 & , 2 \leq t < 4 \\ 0 & , t \geq 4 \end{cases}$$

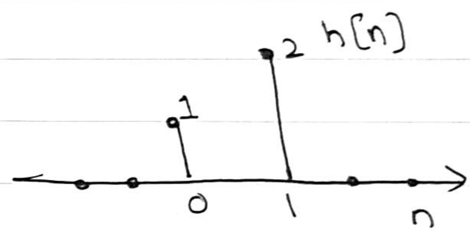


PROBLEM #13:

Convolve the following two functions:

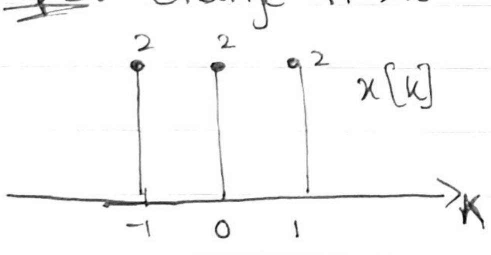


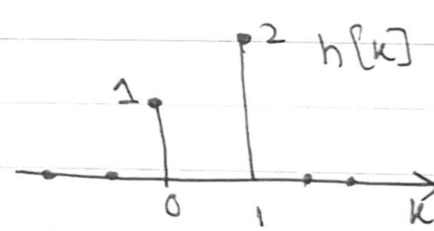
$*$



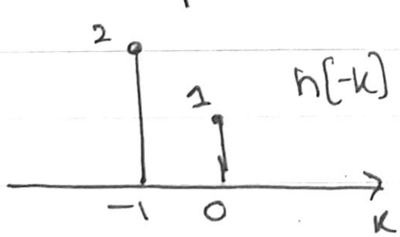
Sol:

Step 1: Change $n \to k$





Step 2: Flip $h(k)$ and ~~or~~ shift



$\Rightarrow$