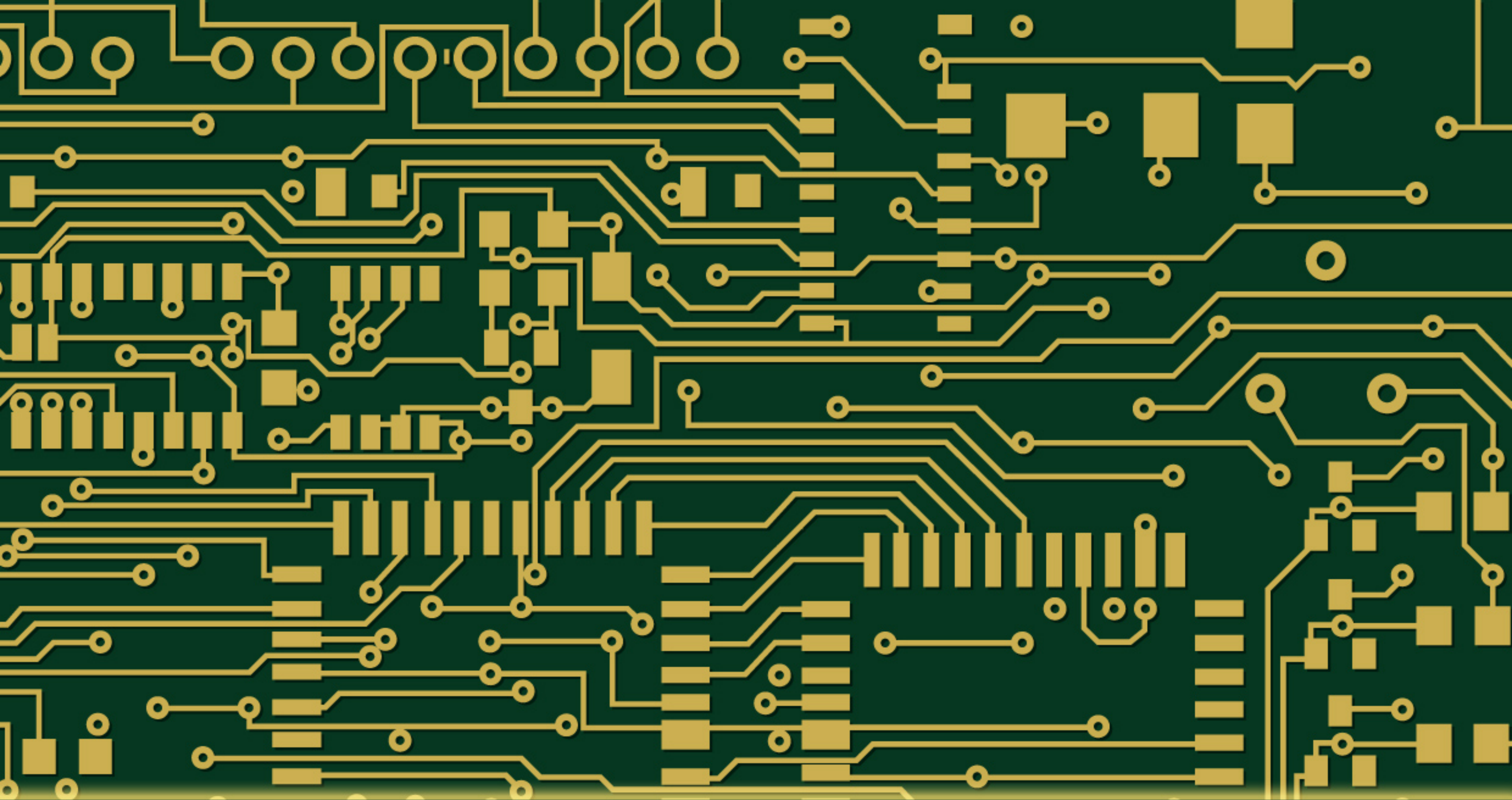
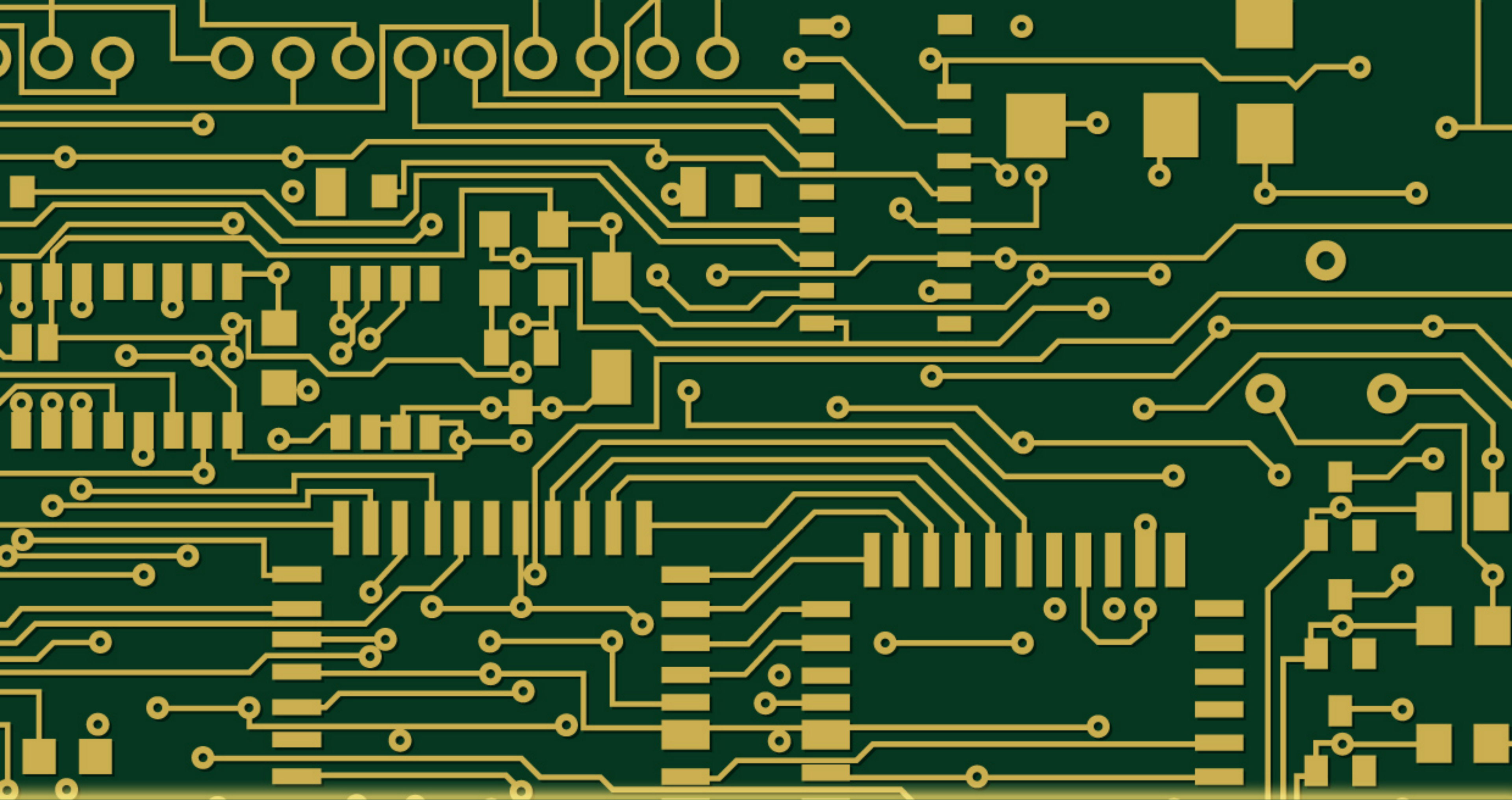


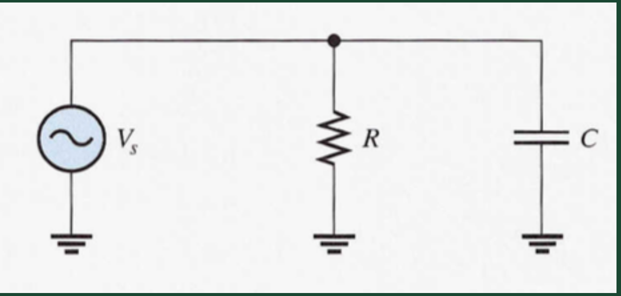
Circuit Analysis-II



Impedance & Admittance

Impedance of Parallel RC Circuits

- ✓ A basic parallel RC circuit connected to an AC voltage source is shown below:



- ✓ Since there are only two components, R and C, the total impedance can be found from the product-over-sum rule:

$$Z = \frac{(R\angle 0^\circ)(X_C\angle -90^\circ)}{R - jX_C}$$

$$Z = \frac{(RX_C)\angle(0^\circ - 90^\circ)}{\sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)}$$

Impedance of Parallel RC Circuits (cont.)

- ✓ By dividing the magnitude expression in the numerator by that in the denominator and by subtracting the angle in the denominator from that in the numerator, you get:

$$Z = \left(\frac{RX_C}{\sqrt{R^2 + X_C^2}}\right)\angle\left(-90^\circ + \tan^{-1}\left(\frac{X_C}{R}\right)\right)$$

- ✓ Equivalently, this expression can be written as:

$$Z = \left(\frac{RX_C}{\sqrt{R^2 + X_C^2}}\right)\angle\left(-\tan^{-1}\left(\frac{X_C}{R}\right)\right)$$

Admittance of Parallel RC Circuits

- ✓ Conductance, **G**, is the reciprocal of resistance. The phasor expression is expressed as:

$$\bar{G} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$$

- ✓ Capacitive susceptance (**B$_C$**) is the reciprocal of capacitive reactance. The phasor expression for capacitive susceptance is:

$$\bar{B}_C = \frac{1}{X_C\angle -90^\circ} = B_C\angle 90^\circ = +jB_C$$

- ✓ Admittance (**Y**) is the reciprocal of impedance:

$$\bar{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle \mp\theta$$

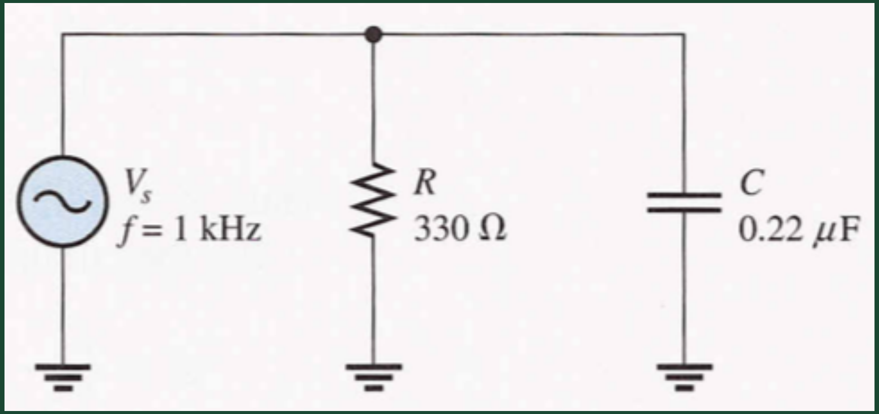
Admittance of Parallel RC Circuits (cont.)

- ✓ The unit of each of these terns is the siemens (S), which is the reciprocal of the ohm.
- ✓ In working with parallel circuits, it is often easier to use conductance (**G**), capacitive susceptance (**B_C**) and admittance (**B**) rather than resistance (R), capacitive reactance (X_C) and impedance (Z).
- ✓ In a parallel RC circuit the total admittance is simply the phasor sum of the conductance and capacitive susceptance:

$$\bar{Y} = G + jB_C$$

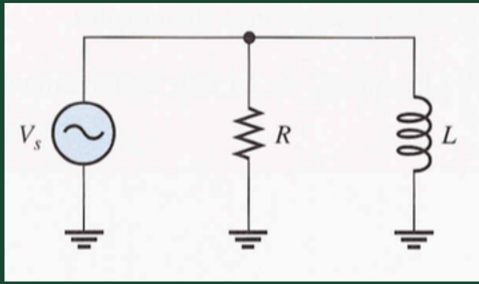
Example #1

- ✓ Determine the total admittance (**Y**) and then convert it to total impedance (**Z**) in figure shown below. Draw the admittance phasor diagram:



Impedance of Parallel RL Circuits

- ✓ A basic parallel RL circuit connected to an AC voltage source is shown below:



- ✓ The expression for the total impedance of a two-component parallel RL circuit is as follows:

$$\bar{Z} = \left(\frac{R X_L}{\sqrt{R^2 + X_L^2}} \right) \angle \tan^{-1}\left(\frac{R}{X_L} \right)$$

Admittance of Parallel RL Circuits

- ✓ For parallel RL circuits, the phasor expression for inductive susceptance (**B**$_L$) is:

$$\bar{B}_L = \frac{1}{X_L\angle 90^\circ} = B_L\angle -90^\circ = -jB_L$$

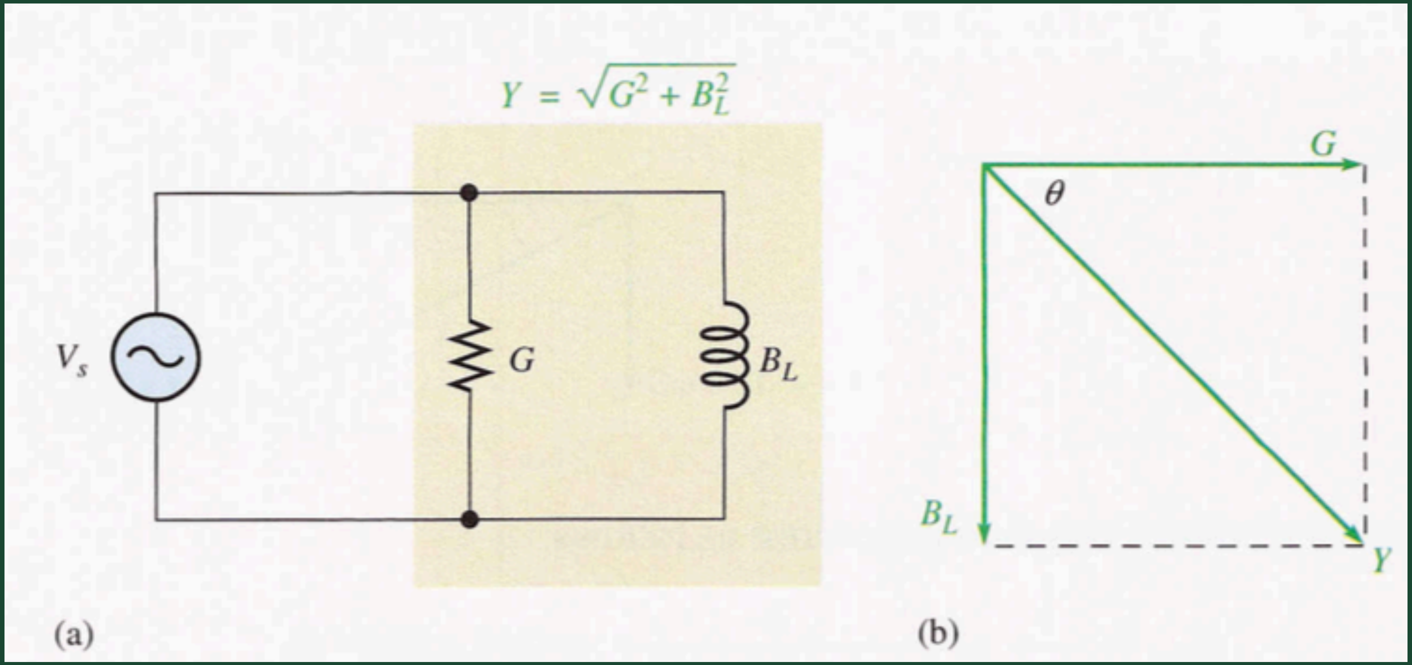
- ✓ The phasor expression for admittance is:

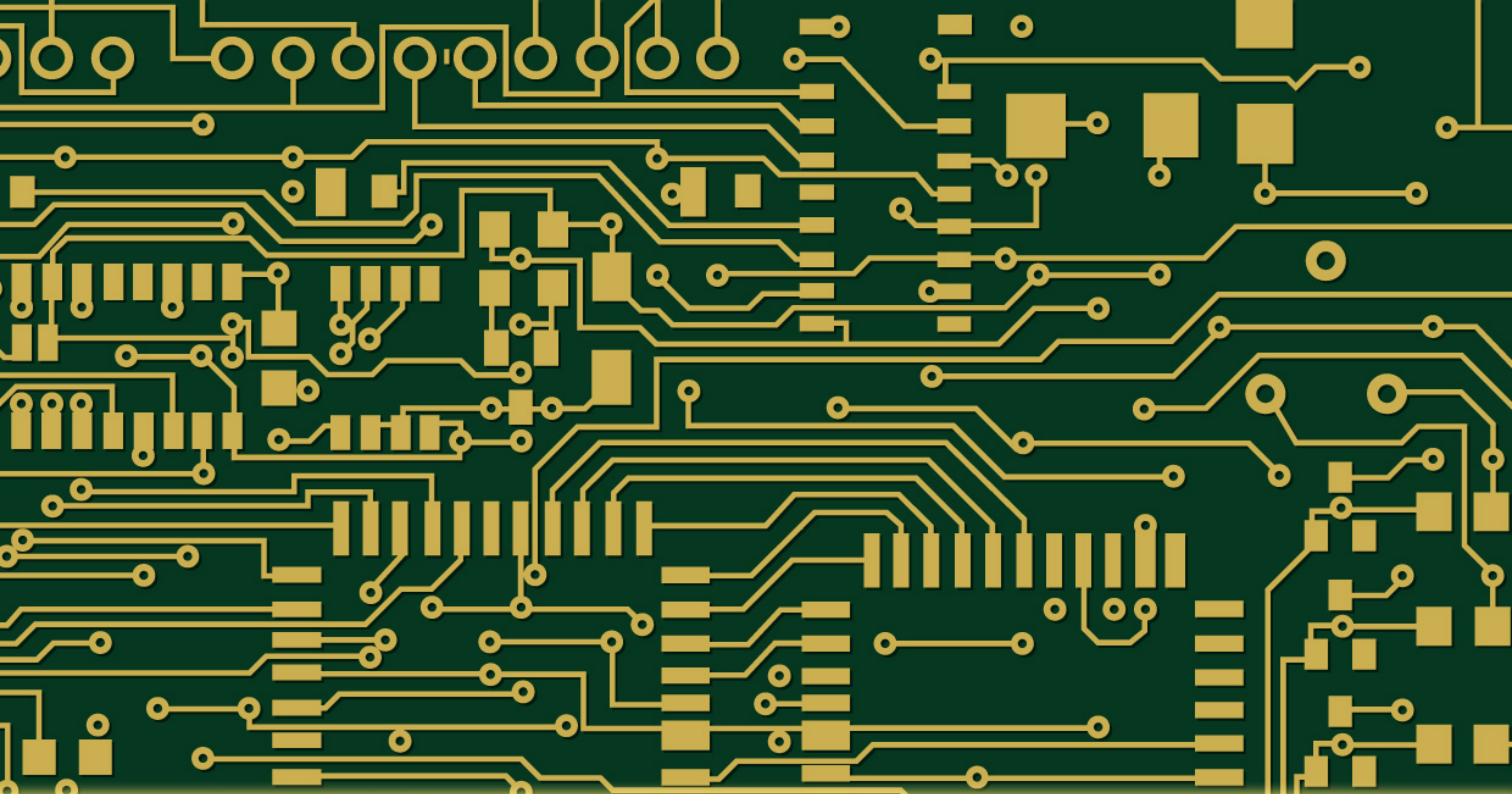
$$\bar{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle \mp\theta$$

- ✓ The total impedance is the phasor sum of the conductance and the inductive susceptance:

$$\bar{Y} = G - jB_L$$

Admittance of Parallel RL Circuits (cont.)

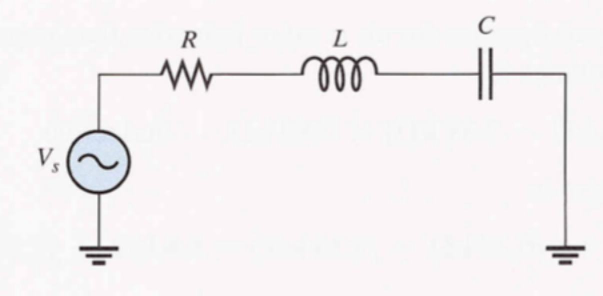




RLC Series & Parallel Circuits

Impedance of Series RLC Circuits

- ✓ A series RLC circuit is shown below:



- ✓ Inductance reactance (X_L) causes the total current to lag the applied voltage.
- ✓ Capacitive reactance (X_C) has the opposite effect: it causes the current to lead the voltage.
- ✓ Thus X_L and X_C tend to offset each other, when they are equal they cancel and the total reactance is zero. $X_{tot} = |X_L - X_C|$.

Impedance of Series RLC Circuits (cont.)

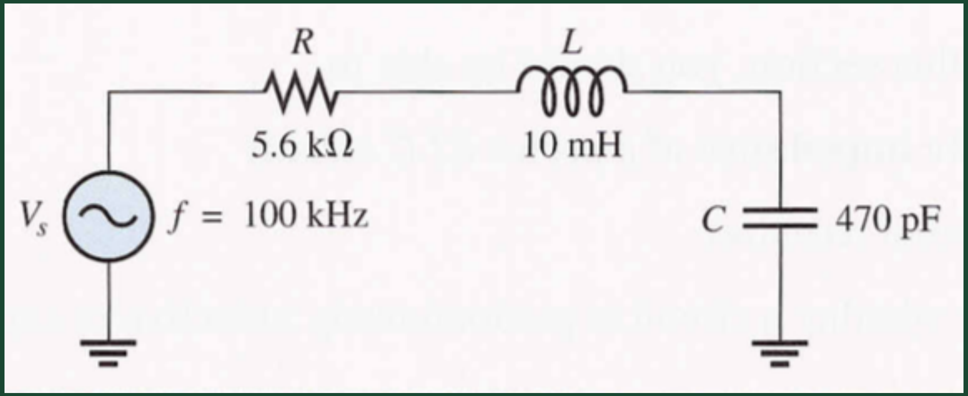
- ✓ When $X_L > X_C$ the circuit is predominantly inductive, and when $X_C > X_L$ the circuit is predominantly capacitive.
- ✓ The total impedance for the series RLC circuit is stated in rectangular and polar form as follows:

$$\bar{Z} = R + jX_L - jX_C$$

$$\bar{Z} = \sqrt{R^2 + \left(X_L - X_C\right)^2} \angle \pm \tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

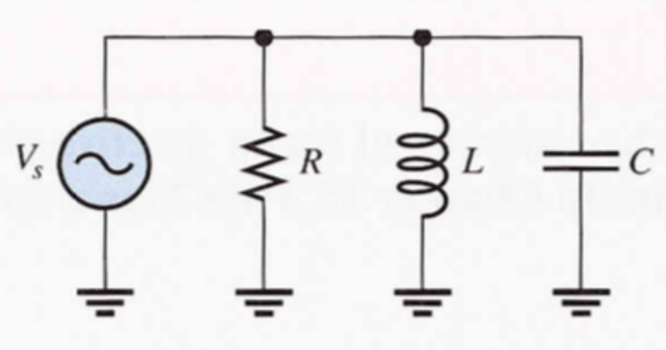
Example #2

- ✓ For the series RLC circuit in the figure below, determine the total impedance. Express it in both rectangular and polar forms:



Impedance of Parallel RLC Circuits

- ✓ The parallel RLC circuit is shown below:



- ✓ The total impedance for RLC circuit is as below:

$$\frac{1}{\bar{Z}} = \frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ}$$

$$\bar{Z} = \frac{1}{\frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ}}$$

Admittance of Parallel RLC Circuits

- ✓ The formulas of conductance (G), capacitive susceptance (B_C), inductive susceptance (B_L) and admittance (Y) are as follows:

$$\bar{G} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$$

$$\bar{B}_C = \frac{1}{X_C\angle -90^\circ} = B_C < 90^\circ = jB_C$$

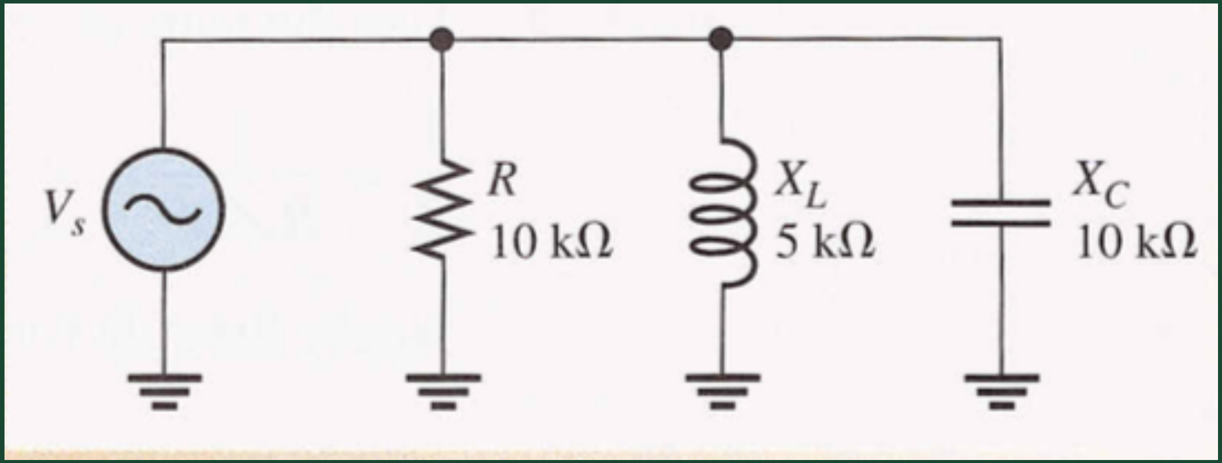
$$\bar{B}_L = \frac{1}{X_L\angle 90^\circ} = B_L < -90^\circ = -jB_L$$

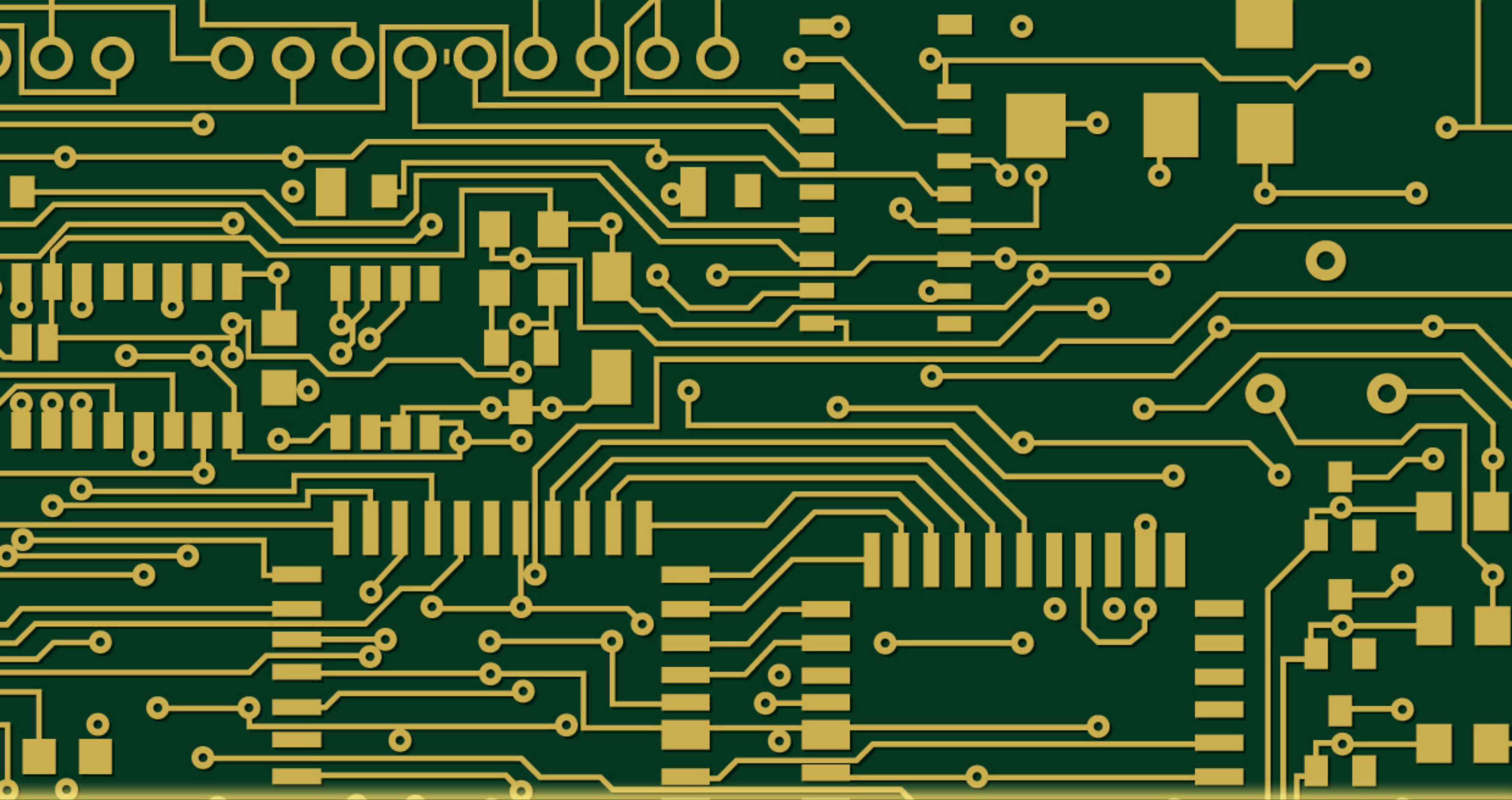
$$\bar{Y} = \frac{1}{Z\angle \pm\theta} = \bar{Y}\angle \mp\theta = G + jB_C - jB_L$$

- ✓ Unit of each of these quantities is the siemens (S).

Example #3

✓ For the RLC circuit in figure below, determine the conductance, capacitive susceptamce, inductive susceptance and total admittance. Also, determine the impedance.





Analysis of Parallel Circuits

Parallel RC Circuits

- ✓ In the analysis of parallel circuits, the Ohm's law formulas using impedance can be rewritten for admittance using the relation Y= 1/Z.
- ✓ The formulas are:

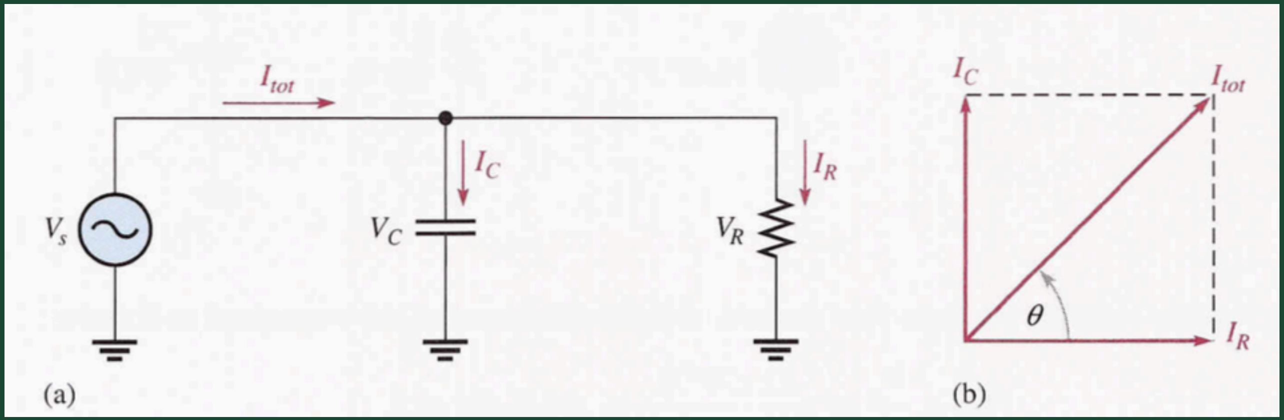
$$\bar{V} = \frac{\bar{I}}{\bar{Y}}$$

$$\bar{I} = \bar{V}\bar{Y}$$

$$\bar{Y} = \frac{\bar{I}}{\bar{V}}$$

Phase Relationships of Current & Voltages

✓ The currents in a basic parallel RC circuit is shown below:



✓ The total current I_{tot}, divides at the junction into the two branch currents I_R and I_C.

✓ The applied voltage V_s, appears across both the resistive and the capacitive branches, so V_s, V_R and V_C are all in phase and of the same magnitude.

Phase Relationships of Current & Voltages (cont.)

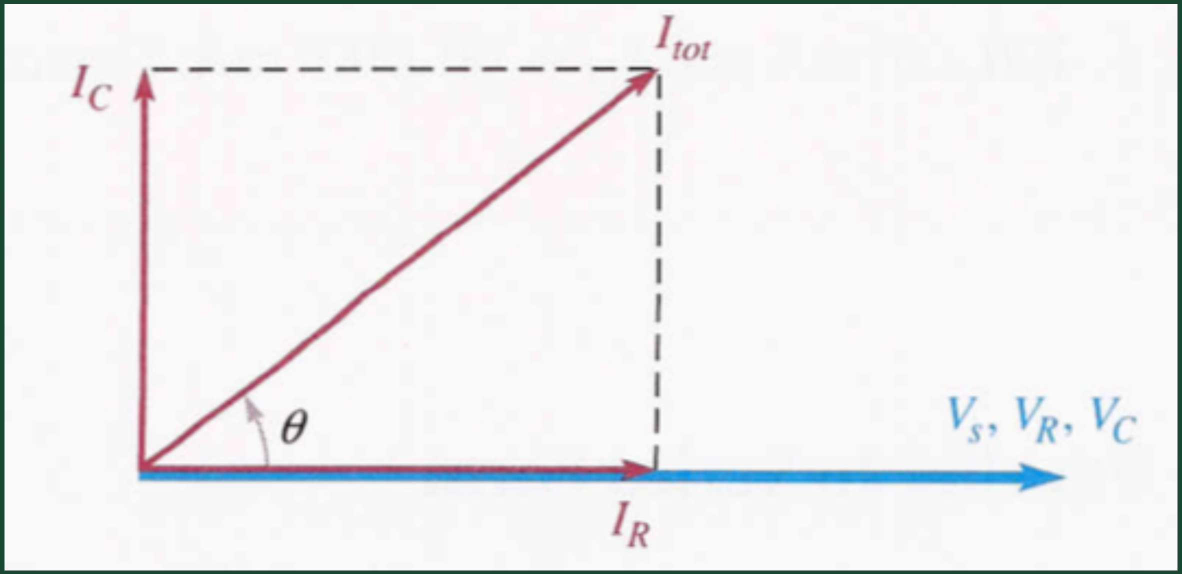
- ✓ The current through the resistor is in phase with the voltage.
- ✓ The current through the capacitor leads the voltage and thus the resistive current by 90°.
- ✓ By Kirchhoff's current law, the total current is the phasor sum of the two branch currents.
- ✓ The total current is expressed as:

$$\overline{I}_{tot} = \overline{I}_R + j\overline{I}_C$$

$$\overline{I}_{tot} = \sqrt{I_R^2 + I_C^2} \angle \tan^{-1}\left(\frac{I_C}{I_R}\right)$$

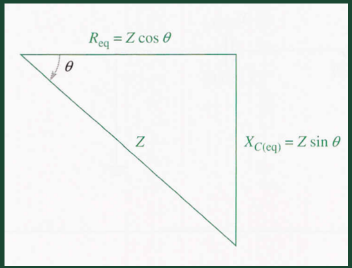
Phase Relationships of Current & Voltages (cont.)

- ✓ Below is the complete current and voltage phasor diagram:



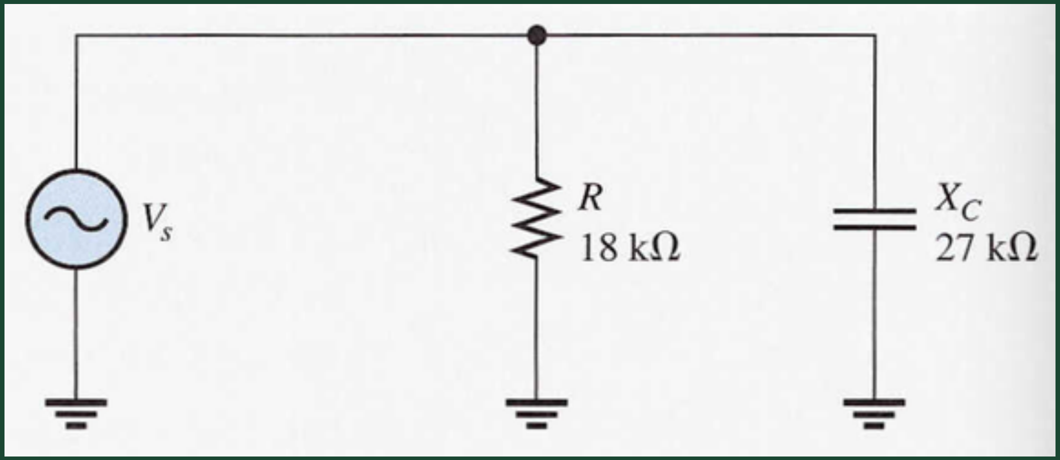
Conversion From Parallel to Series

- ✓ For every parallel RC circuit there is an equivalent series RC circuit for a given frequency.
- ✓ Two circuits are considered equivalent when they both present an equal impedance at their terminals, i.e., the magnitude of impedance and the phase angle are identical.
- ✓ To obtain the equivalent series circuit for a given parallel RC circuit, first find the impedance and phase angle of the parallel circuit.
- ✓ Then use the values of Z and θ to construct the impedance triangle.



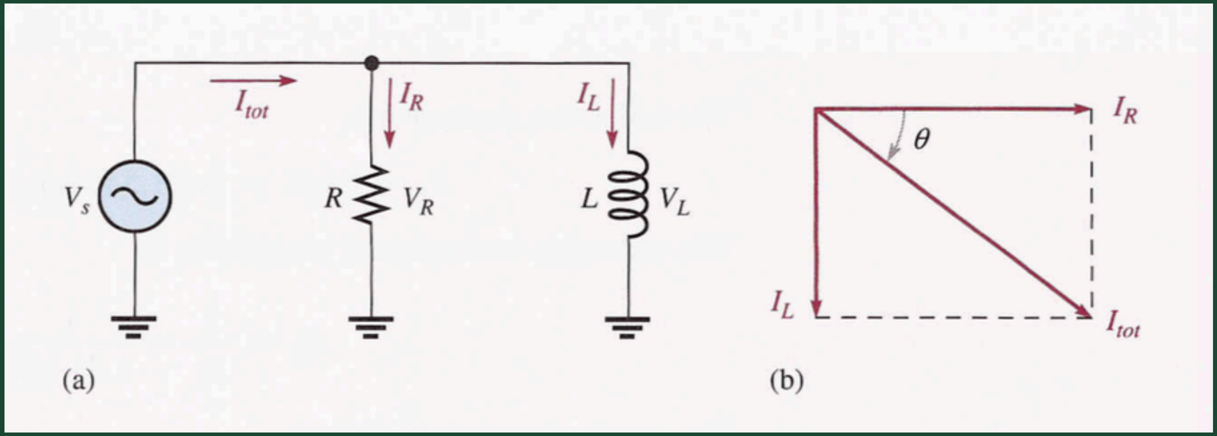
Example #4

✓ Convert the parallel circuit shown below to a series form:



Parallel RL Circuits

- ✓ <u>Phase Relationships of Currents & Voltages</u>:
- ✓ The basic parallel RL circuit is shown below:



- ✓ The current through the resistor is in phase with the voltage.
- ✓ The current through the inductor lags the voltage and the resistor current by 90°.

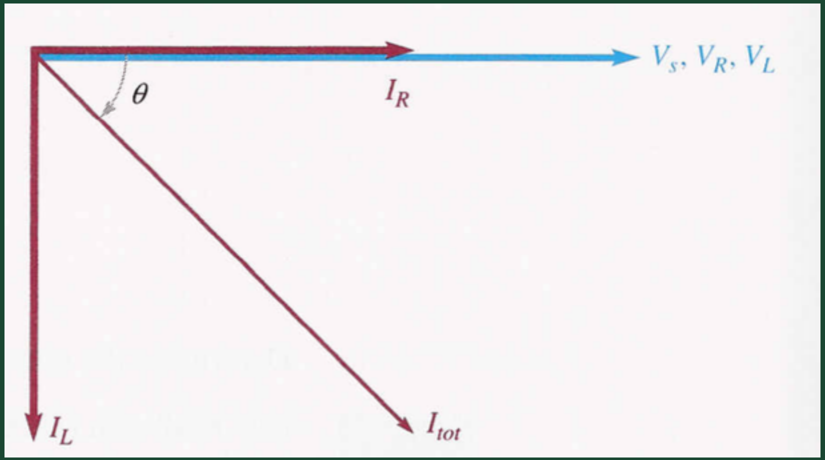
Parallel RL Circuits (cont.)

- ✓ The total current is the phasor sum of the two branches. The total current is expressed as:

$$\overline{I}_{tot} = \overline{I}_R - j\overline{I}_L$$

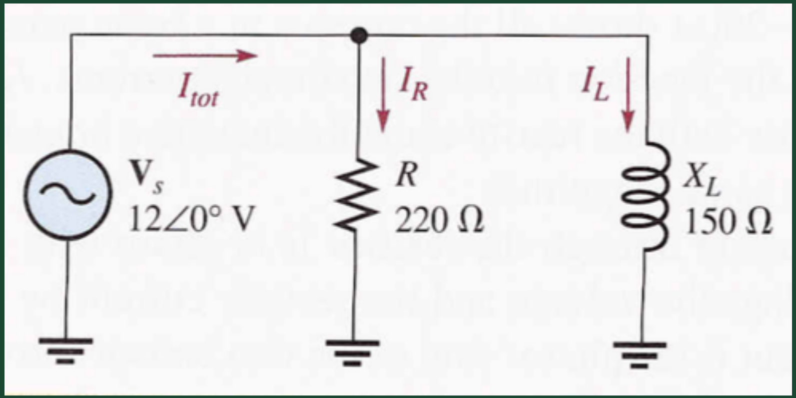
$$\overline{I}_{tot} = \sqrt{I_R^2 + I_L^2} \angle -\tan^{-1}\left(\frac{I_L}{I_R}\right)$$

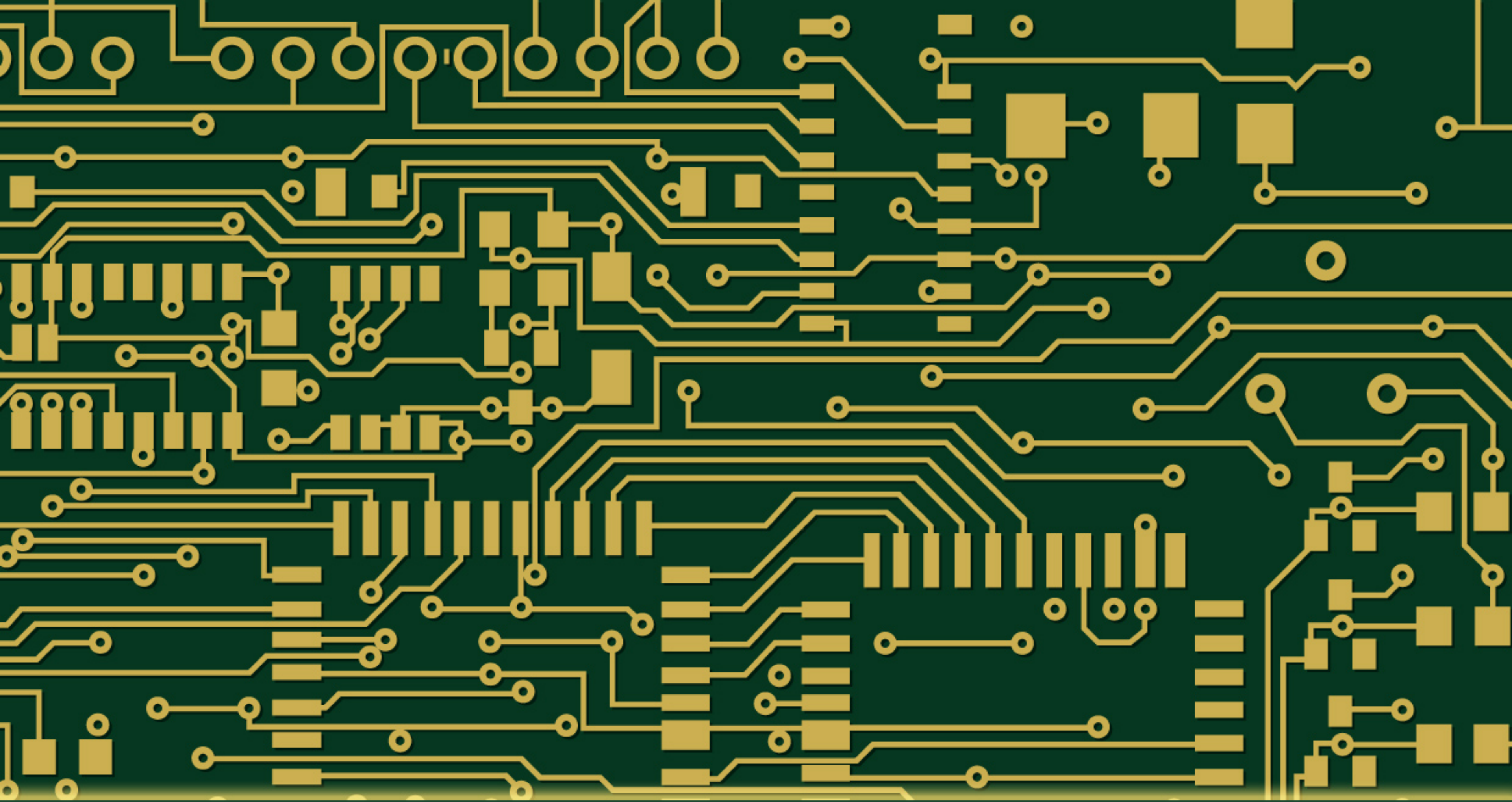
- ✓ The phasor diagram for the current and voltage are shown below:



Example #5

✓ Determine the value of each current in figure below and describe the phase relationship of each with the applied voltage. Draw the current phasor diagram.





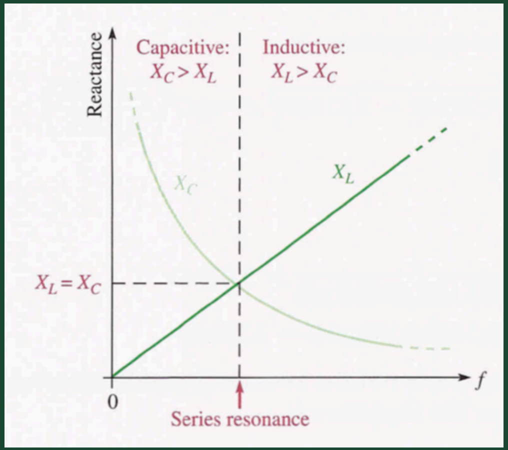
Analysis of RLC Circuits

Series RLC Circuits

- ✓ The total reactance in RLC circuit behaves as follows:
  - ✓ Starting at very low frequency, X_C is high and X_L is low and the circuit is predominantly capacitive.
  - ✓ As the frequency increased, X_C decreases and X_L increases until a value is reached where $X_C=X_L$ and the two reactance cancel making the circuit purely resistive.
  - ✓ This condition is series resonance.
  - ✓ As the frequency is increased further X_L becomes greater than X_C and the circuit is predominantly inductive.

Series RLC Circuits (cont.)

✓ As shown below:

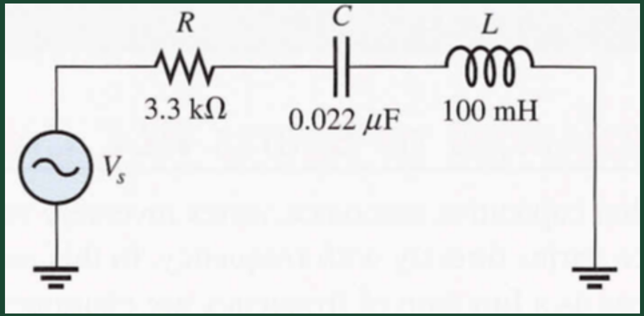


✓ In series RLC circuit the capacitor voltage and the inductor voltage are always 180° out of phase with each other.

✓ For this reason V_C and V_L subtract from each other and thus the voltage across L and C combined is always less than the larger individual voltage across either element.

Example #6

- ✓ For the following input frequencies, find the impedance in polar form for the circuit shown below.



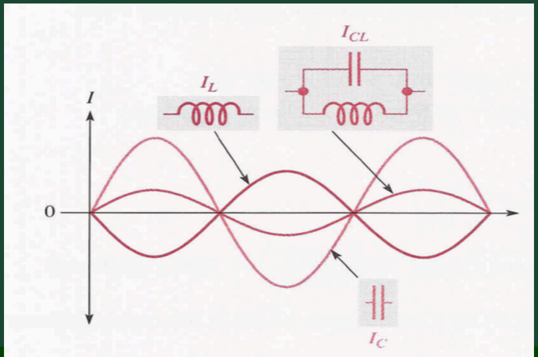
- ✓ (a): f = 1kHz
- ✓ (b): f = 3.5kHz

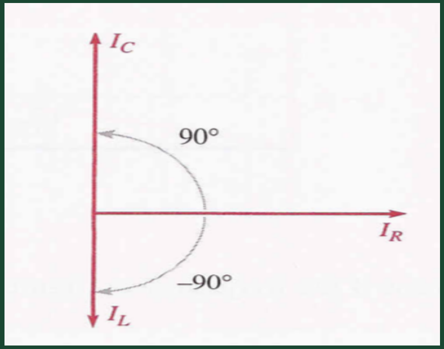
Parallel RLC Circuits

- ✓ As we know the capacitive reactance varies inversely with frequency and that inductive reactance varies directly with frequency.
- ✓ In a parallel RLC circuit at low frequencies the inductive reactance is less than the capacitive reactance, therefore the circuit is inductive.
- ✓ As the frequency is increased X_L increases and X_C decreased until a value is reached where X_L=X_C. This is the point of parallel resonance.
- ✓ As the frequency is increased further, X_C becomes smaller than X_L and the circuit becomes capacitive.

Parallel RLC Circuits (cont.)

- ✓ The current in the capacitive branch and the current in the inductive branch are always 180° out of phase with each other.
- ✓ The total current is actually the difference in their magnitudes.
- ✓ Thus the total current into the parallel branches of L and C is always less than the largest individual branch current.
- ✓ The current in the resistive branch is always 90° out of phase with both reactive currents.





Parallel RLC Circuits (cont.)

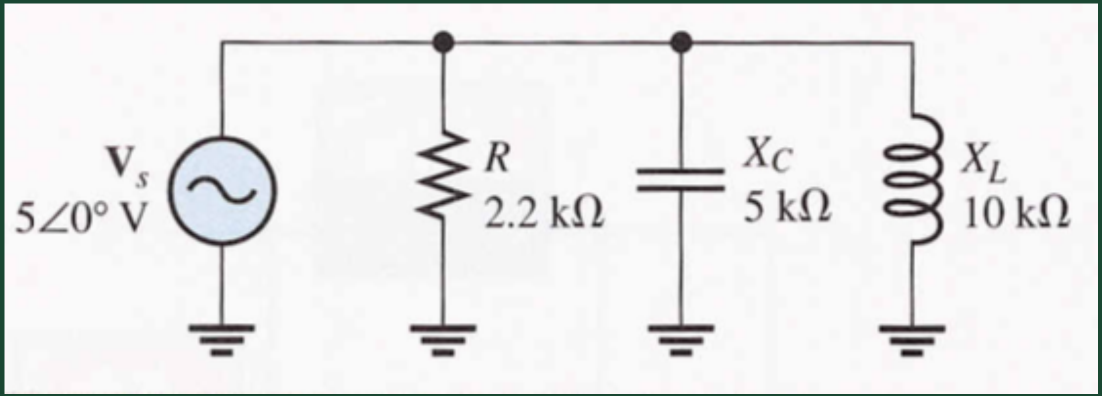
- ✓ The total current can be expressed as:

$$\bar{I}_{tot} = \sqrt{I_R^2 + \left(I_C - I_L\right)^2} \angle \tan^{-1}\left(\frac{I_{CL}}{I_R}\right)$$

- ✓ Where I_{CL} is I_C-I_L the total current into the L and C branches.

Example #7

✓ For the circuit shown below, find each branch current and the total current. Draw a diagram of their relationship.



Thank You