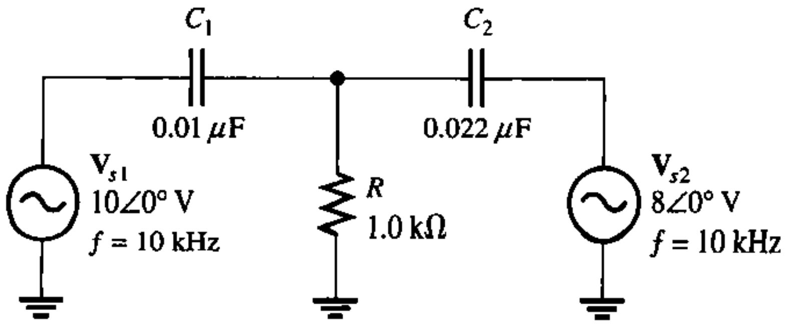
Example #1

Find the current in R of Figure 19–1 using the superposition theorem. Assume the internal source impedances are zero.



Step 1. Replace V_{s2} with its internal impedance (zero in this case), and find the current in R due to V_{s1}, as indicated in Figure 19–2.

$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi(10\text{ kHz})(0.01\ \mu\text{F})} = 1.59\text{ k}\Omega$$

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi(10\text{ kHz})(0.022\ \mu\text{F})} = 723\ \Omega$$

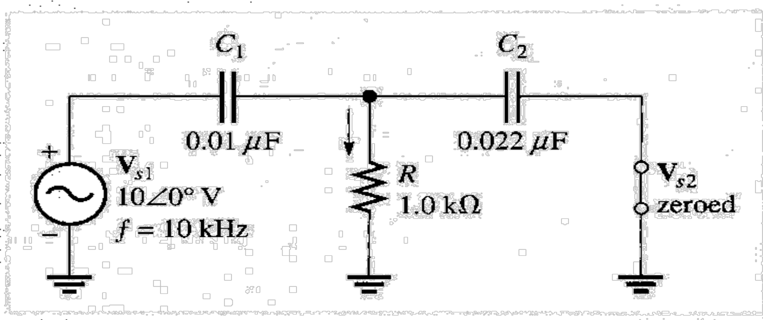


FIGURE 19–2

Looking from V_{s1}, the impedance is

$$\mathbf{Z} = \mathbf{X}_{C1} + \frac{\mathbf{R}\mathbf{X}_{C2}}{\mathbf{R} + \mathbf{X}_{C2}} = 1.59\angle -90^\circ\text{ k}\Omega + \frac{(1.0\angle 0^\circ\text{ k}\Omega)(723\angle -90^\circ\ \Omega)}{1.0\text{ k}\Omega - j723\ \Omega}$$

$$= 1.59\angle -90^\circ\text{ k}\Omega + 588\angle -54.1^\circ\ \Omega$$

$$= -j1.59\text{ k}\Omega + 345\ \Omega - j476\ \Omega = 345\ \Omega - j2.07\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = 2.10\angle -80.5^\circ\text{ k}\Omega$$

$$\mathbf{Z} = 2.10\angle -80.5^\circ\ \text{k}\Omega$$

The total current from V_{s1} is

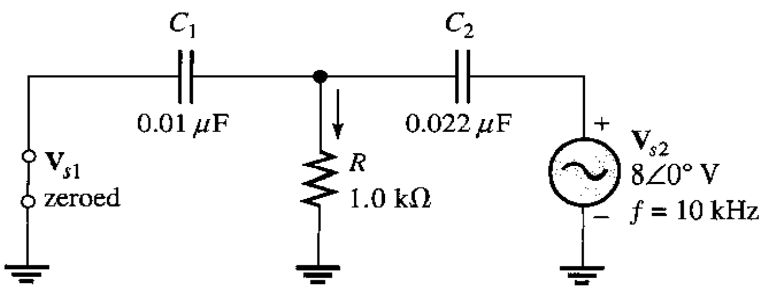
$$\mathbf{I}_{s1} = \frac{\mathbf{V}_{s1}}{\mathbf{Z}} = \frac{10\angle 0^\circ\ \text{V}}{2.10\angle -80.5^\circ\ \text{k}\Omega} = 4.76\angle 80.5^\circ\ \text{mA}$$

Use the current-divider formula. The current through R due to V_{s1} is

$$\mathbf{I}_{R1} = \left(\frac{X_{C2}\angle -90^\circ}{R - jX_{C2}}\right)\mathbf{I}_{s1} = \left(\frac{723\angle -90^\circ\ \Omega}{1.0\ \text{k}\Omega - j723\ \Omega}\right)4.76\angle 80.5^\circ\ \text{mA}$$

$$= (0.588\angle -54.9^\circ\ \Omega)(4.76\angle 80.5^\circ\ \text{mA}) = 2.80\angle 25.6^\circ\ \text{mA}$$

Step 2. Find the current in R due to source V_{s2} by replacing V_{s1} with its internal impedance (zero), as shown in Figure 19–3.



▲ **FIGURE 19–3**

Looking from V_{s2}, the impedance is

$$\mathbf{Z} = \mathbf{X}_{C2} + \frac{\mathbf{R}\mathbf{X}_{C1}}{\mathbf{R} + \mathbf{X}_{C1}} = 723\angle -90^\circ\ \Omega + \frac{(1.0\angle 0^\circ\ \text{k}\Omega)(1.59\angle -90^\circ\ \text{k}\Omega)}{1.0\ \text{k}\Omega - j1.59\ \text{k}\Omega}$$
$$= 723\angle -90^\circ\ \Omega + 847\angle -32.2^\circ\ \Omega$$
$$= -j723\ \Omega + 717\ \Omega - j451\ \Omega = 717\ \Omega - j1174\ \Omega$$

Converting to polar form yields

$$\mathbf{Z} = 1376\angle -58.6^\circ\ \Omega$$

The total current from V_{s2} is

$$\mathbf{I}_{s2} = \frac{\mathbf{V}_{s2}}{\mathbf{Z}} = \frac{8\angle 0^\circ\ \text{V}}{1376\angle -58.6^\circ\ \Omega} = 5.81\angle 58.6^\circ\ \text{mA}$$

Use the current-divider formula. The current through R due to V_{s2} is

$$\mathbf{I}_{R2} = \left(\frac{X_{C1}\angle -90^\circ}{R - jX_{C1}}\right)\mathbf{I}_{s2}$$
$$= \left(\frac{1.59\angle -90^\circ\ \text{k}\Omega}{1.0\ \text{k}\Omega - j1.59\ \text{k}\Omega}\right)5.81\angle 58.6^\circ\ \text{mA} = 4.91\angle 26.4^\circ\ \text{mA}$$

Step 3. Convert the two individual resistor currents to rectangular form and add to get the total current through R.

$$\mathbf{I}_{R1} = 2.80\angle 25.6^\circ\ \text{mA} = 2.53\ \text{mA} + j1.21\ \text{mA}$$
$$\mathbf{I}_{R2} = 4.91\angle 26.4^\circ\ \text{mA} = 4.40\ \text{mA} + j2.18\ \text{mA}$$
$$\mathbf{I}_{R} = \mathbf{I}_{R1} + \mathbf{I}_{R2} = 6.93\ \text{mA} + j3.39\ \text{mA} = \mathbf{7.71\angle 26.1^\circ\ mA}$$

Example #2:

Refer to Figure 19–15. Determine $\mathbf{V}_{th}$ for the circuit within the beige box as viewed from terminals A and B.

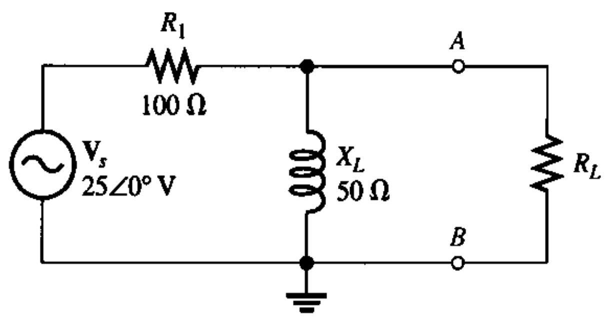


FIGURE 19–15

Solution Remove R_L and determine the voltage from A to B ($\mathbf{V}_{th}$). In this case, the voltage from A to B is the same as the voltage across X_L. This is determined using the voltage-divider method.

$$\mathbf{V}_L = \left(\frac{X_L\angle 90^\circ}{R_1 + jX_L}\right)\mathbf{V}_s$$

$$= \left(\frac{50\angle 90^\circ\ \Omega}{100\ \Omega + j50\ \Omega}\right)$$

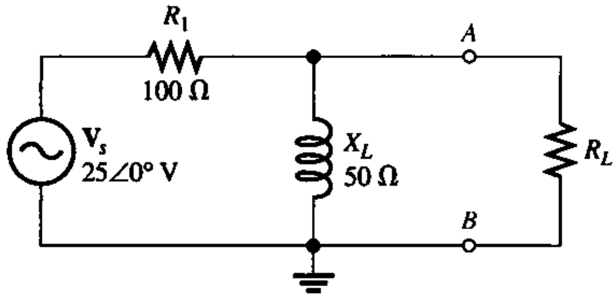
$$= \left(\frac{50\angle 90^\circ\ \Omega}{112\angle 26.6^\circ\ \Omega}\right)25\angle 0^\circ\text{ V} = 11.2\angle 63.4^\circ\text{ V}$$

$$\mathbf{V}_{th} = \mathbf{V}_{AB} = \mathbf{V}_L = \mathbf{11.2\angle 63.4^\circ\ V}$$

Example #3:

Find $\mathbf{Z}_{th}$ for the part of the circuit in Figure 19–18 that is within the beige box as viewed from terminals A and B. This is the same circuit used in Example 19–4.

FIGURE 19–18

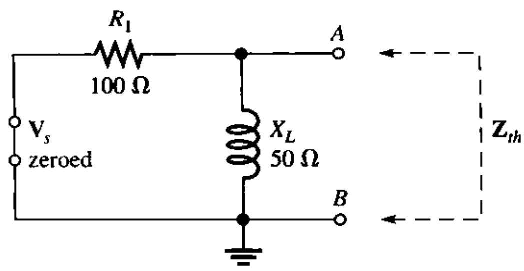


Solution First, replace $\mathbf{V}_s$ with its internal impedance (zero in this case), as shown in Figure 19–19. Looking in between terminals A and B, R_1 and X_L are in parallel. Thus,

$$\mathbf{Z}_{th} = \frac{(R_1\angle 0°)(X_L\angle 90°)}{R_1 + jX_L} = \frac{(100\angle 0°\ \Omega)(50\angle 90°\ \Omega)}{100\ \Omega + j50\ \Omega}$$

$$= \frac{(100\angle 0°\ \Omega)(50\angle 90°\ \Omega)}{112\angle 26.6°\ \Omega} = \mathbf{44.6\angle 63.4°\ \Omega}$$

FIGURE 19–19



Example #4:

Refer to Figure 19–24. Draw the Thevenin equivalent for the circuit within the beige box as viewed from terminals *A* and *B*. This is the circuit used in Examples 19–4 and 19–7.

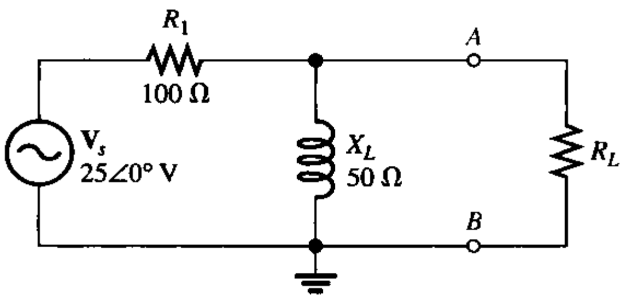


FIGURE 19–24

Solution From Examples 19–4 and 19–7, respectively, $\mathbf{V}_{th} = 11.2\angle 63.4°$ V and $\mathbf{Z}_{th} = 44.6\angle 63.4°\ \Omega$. In rectangular form, the impedance is

$$\mathbf{Z}_{th} = 20\ \Omega + j40\ \Omega$$

This form indicates that the impedance is a 20 Ω resistor in series with a 40 Ω inductive reactance. The Thevenin equivalent circuit is shown in Figure 19–25.

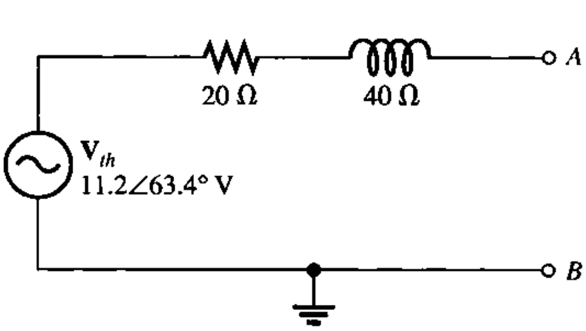
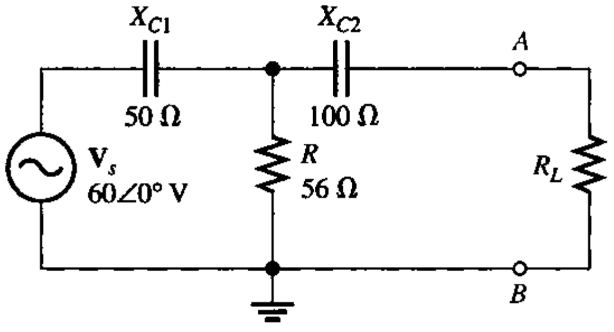


FIGURE 19–25

Example #5:

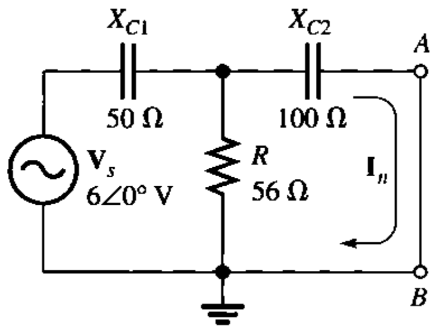
In Figure 19–33, determine $\mathbf{I}_n$ for the circuit as "seen" by the load resistor. The beige area identifies the portion of the circuit to be nortonized.

FIGURE 19–33



Solution Short the terminals *A* and *B*, as shown in Figure 19–34.

FIGURE 19–34



$\mathbf{I}_n$ is the current through the short and is calculated as follows. First, the total impedance viewed from the source is

$$\mathbf{Z} = \mathbf{X}_{C1} + \frac{\mathbf{R}\mathbf{X}_{C2}}{\mathbf{R} + \mathbf{X}_{C2}} = 50\angle -90^\circ\ \Omega + \frac{(56\angle 0^\circ\ \Omega)(100\angle -90^\circ\ \Omega)}{56\ \Omega - j100\ \Omega}$$

$$= 50\angle -90^\circ\ \Omega + 48.9\angle -29.3^\circ\ \Omega$$

$$= -j50\ \Omega + 42.6\ \Omega - j23.9\ \Omega = 42.6\ \Omega - j73.9\ \Omega$$

Converting to polar form yields

$$\mathbf{Z} = 85.3\angle -60.0^\circ\ \Omega$$

Next, the total current from the source is

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{6\angle 0^\circ \text{ V}}{85.3\angle -60.0^\circ \ \Omega} = 70.3\angle 60.0^\circ \text{ mA}$$

Finally, apply the current-divider formula to get $\mathbf{I}_n$ (the current through the short between terminals A and B).

$$\mathbf{I}_n = \left(\frac{\mathbf{R}}{\mathbf{R} + \mathbf{X}_{C2}}\right)\mathbf{I}_s = \left(\frac{56\angle 0^\circ \ \Omega}{56 \ \Omega - j100 \ \Omega}\right)70.3\angle 60.0^\circ \text{ mA} = \mathbf{34.4\angle 121^\circ \ mA}$$

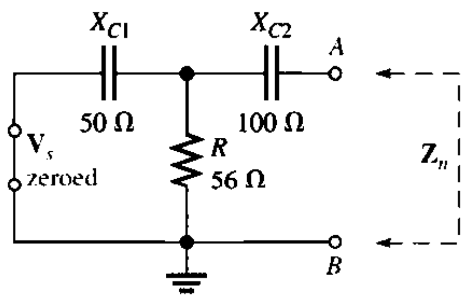
This is the value for the equivalent Norton current source.

Example #6:

Find $\mathbf{Z}_n$ for the circuit in Figure 19–33 (Example 19–13) viewed from the open across terminals A and B.

Solution First, replace $\mathbf{V}_s$ with its internal impedance (zero), as indicated in Figure 19–35.

FIGURE 19–35



Looking in between terminals A and B, C_2 is in series with the parallel combination of R and C_1. Thus,

$$\mathbf{Z}_n = \mathbf{X}_{C2} + \frac{\mathbf{R}\mathbf{X}_{C1}}{\mathbf{R} + \mathbf{X}_{C1}} = 100\angle -90^\circ \ \Omega + \frac{(56\angle 0^\circ \ \Omega)(50\angle -90^\circ \ \Omega)}{56 \ \Omega - j50 \ \Omega}$$

$$= 100\angle -90^\circ \ \Omega + 37.3\angle -48.2^\circ \ \Omega$$

$$= -j100 \ \Omega + 24.8 \ \Omega - j27.8 \ \Omega = \mathbf{24.8 \ \Omega - j128 \ \Omega}$$

The Norton equivalent impedance is a 24.8 Ω resistance in series with a 128 Ω capacitive reactance.