ISRA UNIVERSITY

Islamabad Campus

Program: BTECH (Electrical)
Semester – Spring 2018

ETSS-314 Signal & Systems

Assignment – 3 Solution **Due Date: 29/05/2018**
Marks: 20 **Handout Date: 22/05/2018**

Question # 1:

A particular LTI system is described by the difference equation:

$$y[n]+\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=x[n]-x[n-1]$$

Find the impulse response h [n] of the system. (Discrete time Fourier Transform)

Solution:

The use of the Fourier transform simplifies the analysis of the difference equation:

$$y[n]+\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=x[n]-x[n-1]$$

$$Y(e^{j\omega})+\frac{1}{4}e^{-j\omega}Y(e^{j\omega})-\frac{1}{8}e^{-2j\omega}Y(e^{j\omega})=X(e^{j\omega})-e^{-j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega})\left(1+\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)=X(e^{j\omega})\left(1-e^{-j\omega}\right)$$

$$H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1-e^{-j\omega}}{1+\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}}$$

$$H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1-e^{-j\omega}}{\left(1+\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

Using Partial fraction expansion, we see that:

$$H(e^{j\omega})=\frac{1-e^{-j\omega}}{\left(1+\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}=\frac{A}{\left(1+\frac{1}{2}e^{-j\omega}\right)}+\frac{B}{\left(1-\frac{1}{4}e^{-j\omega}\right)}\rightarrow eq1$$

Cross multiplication yields:

$$1-e^{-j\omega}=A\left(1-\frac{1}{4}e^{-j\omega}\right)+B\left(1+\frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega}=4$, gives:

$$1-4=A\left(1-\frac{1}{4}\times(4)\right)+B\left(1+\frac{1}{2}\times(4)\right)$$
$$1-4=A(0)+B(1+2)$$
$$-3=B(3)\Longrightarrow B=-1$$
$$1=A\left(1-\frac{3}{4}e^{-j\omega}\right)+B\left(1-\frac{1}{2}e^{-j\omega}\right)$$
Putting $e^{-j\omega}=-2$, gives:
$$1-(-2)=A\left(1-\frac{1}{4}\times(-2)\right)+B\left(1+\frac{1}{2}\times(-2)\right)$$
$$1+2=A\left(\frac{2+1}{2}\right)+B(0)$$
$$3=A\left(\frac{3}{2}\right)\Longrightarrow A=2$$
Putting values of A and B in eq(1) gives:
$$H\left(e^{j\omega}\right)=\frac{2}{\left(1+\frac{1}{2}e^{-j\omega}\right)}+\frac{-1}{\left(1-\frac{1}{4}e^{-j\omega}\right)}$$
Taking the inverse Fourier transform, we obtain:
$$h[n]=2\left(-\frac{1}{2}\right)^n u[n]-\left(\frac{1}{4}\right)^n u[n]$$

Question # 2:

Using partial fraction expansion and the fact that:
$$(a)^n u[n]\leftrightarrow\frac{1}{1-az^{-1}}\ ,\ |z|>|a|$$
Find the inverse z-transform of:
$$X(z)=\frac{1-\frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})}\ ,\ |z|>2$$
Solution:

Using partial fraction expansion:
$$X(z)=\frac{1-\frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})}=\frac{A}{(1-z^{-1})}+\frac{B}{(1+2z^{-1})}\rightarrow eq(1)$$
Cross multiplication gives:
$$1-\frac{1}{3}z^{-1}=A(1+2z^{-1})+B(1-z^{-1})$$
Putting $z^{-1}=1$, gives:
$$1-\frac{1}{3}(1)=A\big(1+2(1)\big)+B(1-1)$$
$$\frac{3-1}{3}=A(1+2)+B(0)$$
$$\frac{2}{3}=A(3)\Longrightarrow A=\frac{2}{3}\times\frac{1}{3}=\frac{2}{9}$$

Putting $z^{-1} = -\frac{1}{2}$, gives:

$$1 - \frac{1}{3}\left(-\frac{1}{2}\right) = A\left(1 + 2\left(-\frac{1}{2}\right)\right) + B\left(1 - \left(-\frac{1}{2}\right)\right)$$

$$1 + \frac{1}{6} = A(0) + B\left(1 + \frac{1}{2}\right)$$

$$\frac{6+1}{6} = B\left(\frac{2+1}{2}\right)$$

$$\frac{7}{6} = B\left(\frac{3}{2}\right) \Longrightarrow A = \frac{7}{6} \times \frac{2}{3} = \frac{7}{9}$$

Putting values of A and B in eq(1) gives:

$$X(z) = \frac{\frac{2}{9}}{(1 - z^{-1})} + \frac{\frac{7}{9}}{(1 + 2z^{-1})}$$

Taking the inverse z-transform, we obtain:

$$x[n] = \frac{2}{9}u[n] + \frac{7}{9}(-2)^n u[n]$$

Good Luck