ISRA UNIVERSITY

Islamabad Campus

Program: BTECH (Electrical)

Semester – Spring 2018

ETCA-252 Circuit Analysis-II

Assignment – 4 & 5 **Due Date: 19/06/2018**

Marks: 50 **Handout Date: 29/05/2018**

Question # 1:

Perform the following operations:

1. $(8 + j5) + (2 + j1)$
2. $(3 + j4) - (1 + j2)$
3. $Multiply: (50\angle 10°)(30\angle - 60°)$

(Marks 06)

Solution:

1. $(8 + j5) + (2 + j1)$

   $(8 + j5) + (2 + j1) = (8 + 2) + j(5 + 1) \Rightarrow 10 + j6$

2. $(3 + j4) - (1 + j2)$

   $(3 + j4) - (1 + j2) = (3 - 1) + j(4 - 2) \Rightarrow 2 + j2$

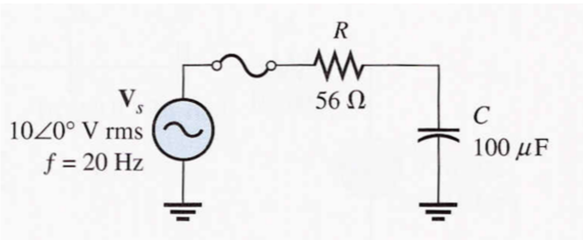
3. $Multiply: (50\angle 10°)(30\angle - 60°)$

$$(50\angle 10°)(30\angle - 60°) = (50)(30)\angle\big(10° + (-60°)\big) \Rightarrow 1500\angle - 50°$$

Question # 2:

For the circuit in figure below, determine the following in polar form:

1. **Z**
2. $\mathbf{I}_{tot}$
3. $\mathbf{V}_R$
4. $\mathbf{V}_C$



(Marks 10)

Solution:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(20Hz)(100\times10^{-6})} \Rightarrow 79.6\Omega$$

1. **Z**

$$\boldsymbol{Z} = \sqrt{R^2 + {X_C}^2}\angle - tan^{-1}\left(\frac{X_C}{R}\right)$$

$$= \sqrt{56\Omega^2 + (79.6\Omega)^2}\angle - tan^{-1}\left(\frac{79.6\Omega}{56\Omega}\right) \Rightarrow 97.32\angle - 55°\Omega$$

2. **I**$_{tot}$

$$\boldsymbol{I}_{tot} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \frac{10\angle0°}{97.32\angle - 55°\Omega}$$

$$= \frac{10}{97.32}\angle\big(0° - (-55°)\big) \Rightarrow 103\angle55°\, mA$$

3. **V**$_R$

$$\boldsymbol{V}_R = \left(\frac{R}{\sqrt{R^2 + {X_C}^2}}\right)\boldsymbol{V}_{in}\angle\emptyset = \left(\frac{56\Omega}{97.32\Omega}\right)(10\angle55°)$$

$$\Rightarrow 5.76\angle 55°\Omega$$

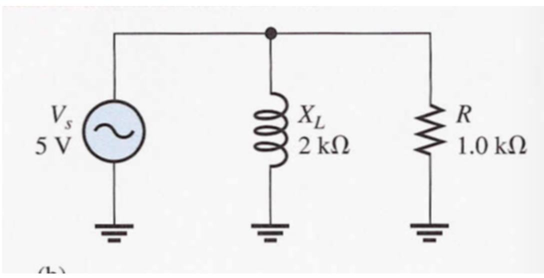
4. **V**$_C$

$$\boldsymbol{V}_R = \left(\frac{X_C}{\sqrt{R^2 + {X_C}^2}}\right)\boldsymbol{V}_{in}\angle\emptyset = \left(\frac{79.6\Omega}{97.32\Omega}\right)\left(10\angle - tan^{-1}\left(\frac{R}{X_C}\right)\right)$$

$$\Rightarrow 8.18\angle - 35.1°\Omega$$

Question # 3:

Determine the magnitude and phase angle of the total impedance:



(Marks 10)

Solution:

The total impedance is:

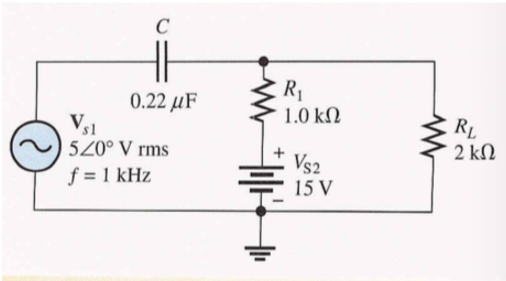
$$\mathbf{Z} = \left(\frac{(1.0\,\text{k}\Omega)(2\,\text{k}\Omega)}{\sqrt{(1.0\,\text{k}\Omega)^2 + (2\,\text{k}\Omega)^2}}\right)\angle\tan^{-1}\left(\frac{1.0\,\text{k}\Omega}{2\,\text{k}\Omega}\right) = 894\angle 26.6^\circ\ \Omega$$

Thus, $Z = 894\ \Omega$ and $\theta = 26.6^\circ$.

The positive angle indicates that the voltage leads the current, as opposed to the RC case where voltage lags the current.

Question # 4:

Find the total current in the load resistor, R_L. Assume the sources are ideal.



(Marks 10)

Solution:

Step 1. Find the current through R_L due to the ac source V_{s1} by zeroing (replacing with its internal impedance) the dc source V_{S2}, as shown in Figure 19–8. Looking from V_{s1}, the impedance is

$$\mathbf{Z} = \mathbf{X}_C + \frac{\mathbf{R}_1\mathbf{R}_L}{\mathbf{R}_1 + \mathbf{R}_L}$$

$$X_C = \frac{1}{2\pi(1.0\,\text{kHz})(0.22\,\mu\text{F})} = 723\ \Omega$$

$$\mathbf{Z} = 723\angle -90^\circ\ \Omega + \frac{(1.0\angle 0^\circ\,\text{k}\Omega)(2\angle 0^\circ\,\text{k}\Omega)}{3\angle 0^\circ\,\text{k}\Omega}$$

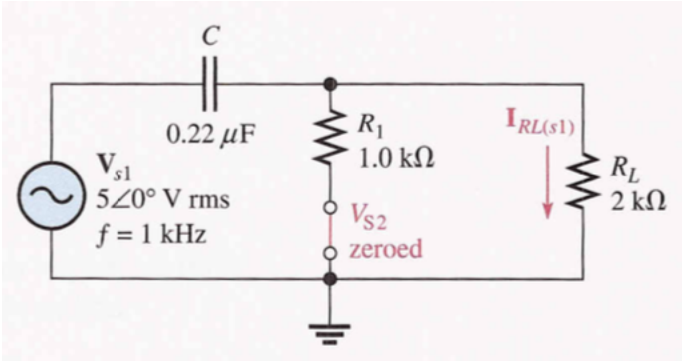
$$= -j723\ \Omega + 667\ \Omega = 984\angle -47.3^\circ\ \Omega$$

The total current from the ac source is

$$\mathbf{I}_{s1} = \frac{\mathbf{V}_{s1}}{\mathbf{Z}} = \frac{5\angle 0^\circ\,\text{V}}{984\angle -47.3^\circ\ \Omega} = 5.08\angle 47.3^\circ\ \text{mA}$$

Use the current-divider approach. The current in R_L due to V_{s1} is

$$\mathbf{I}_{RL(s1)} = \left(\frac{R_1}{R_1 + R_L}\right)\mathbf{I}_{s1} = \left(\frac{1.0\,\text{k}\Omega}{3\,\text{k}\Omega}\right)5.08\angle 47.3^\circ\ \text{mA} = 1.69\angle 47.3^\circ\ \text{mA}$$

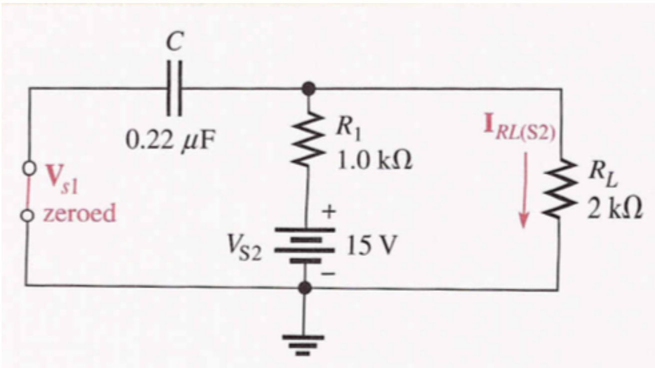


Step 2. Find the current in R_L due to the dc source V_{S2} by zeroing V_{s1} (replacing with its internal impedance), as shown in Figure 19–9. The impedance magnitude as seen by V_{S2} is

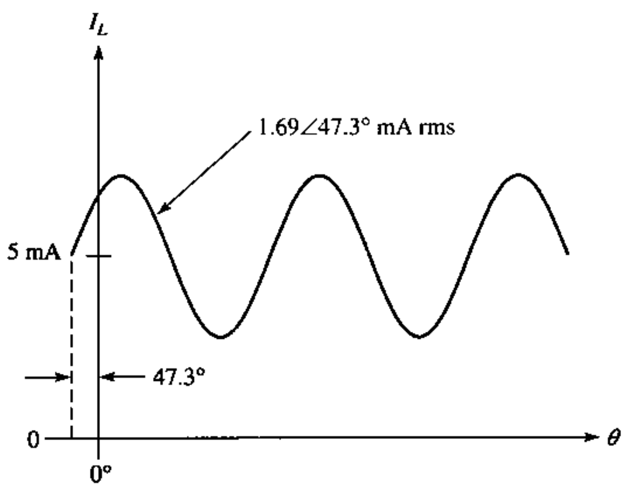
$$Z = R_1 + R_L = 3\,\text{k}\Omega$$

The current produced by V_{S2} is

$$I_{RL(S2)} = \frac{V_{S2}}{Z} = \frac{15\text{ V}}{3\,\text{k}\Omega} = 5\text{ mA dc}$$

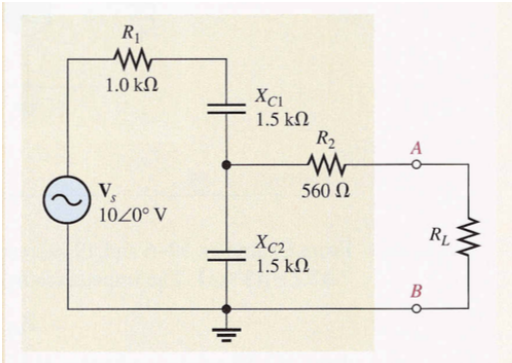


Step 3. By superposition, the total current in R_L is 1.69∠47.3° mA riding on a dc level of 5 mA, as indicated in Figure 19–10.



Question # 5:

Determine V_{th} and Z_{th} for the circuit within the beige box and draw its Thevenin Equivalent.



(Marks 10)

Solution:

Thevenin's voltage for the circuit between terminals *A* and *B* is the voltage that appears across *A* and *B* with R_L removed from the circuit.

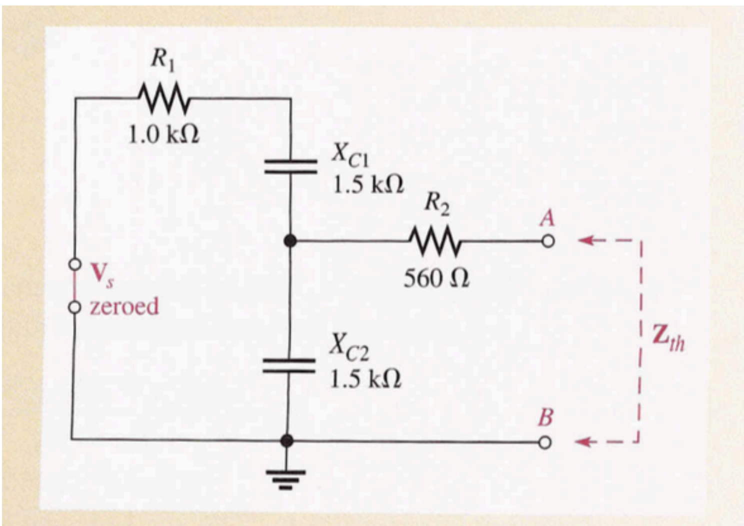
There is no voltage drop across R_2 because the open between terminals *A* and *B* prevents current through it. Thus, $\mathbf{V}_{AB}$ is the same as $\mathbf{V}_{C2}$ and can be found by the voltage-divider formula.

$$\mathbf{V}_{AB} = \mathbf{V}_{C2} = \left(\frac{X_{C2}\angle -90^\circ}{R_1 - jX_{C1} - jX_{C2}}\right)\mathbf{V}_s = \left(\frac{1.5\angle -90^\circ \text{ k}\Omega}{1.0\text{ k}\Omega - j3\text{ k}\Omega}\right)10\angle 0^\circ \text{ V}$$

$$= \left(\frac{1.5\angle -90^\circ \text{ k}\Omega}{3.16\angle -71.6^\circ \text{ k}\Omega}\right)10\angle 0^\circ \text{ V} = 4.75\angle -18.4^\circ \text{ V}$$

$$\mathbf{V}_{th} = \mathbf{V}_{AB} = \mathbf{4.75\angle -18.4^\circ \text{ V}}$$

For calculating Z_{th}, first replace the voltage source with its internal impedance (zero in this case) as shown below:



Looking from terminals A and B, C_2 appears in parallel with the series combination of R_1 and C_1. This entire combination is in series with R_2. The calculation for $\mathbf{Z}_{th}$ is as follows:

$$\mathbf{Z}_{th} = R_2\angle 0^\circ + \frac{(X_{C2}\angle -90^\circ)(R_1 - jX_{C1})}{R_1 - jX_{C1} - jX_{C2}}$$

$$= 560\angle 0^\circ\ \Omega + \frac{(1.5\angle -90^\circ\ \text{k}\Omega)(1.0\ \text{k}\Omega - j1.5\ \text{k}\Omega)}{1.0\ \text{k}\Omega - j3\ \text{k}\Omega}$$

$$= 560\angle 0^\circ\ \Omega + \frac{(1.5\angle -90^\circ\ \text{k}\Omega)(1.8\angle -56.3^\circ\ \text{k}\Omega)}{3.16\angle -71.6^\circ\ \text{k}\Omega}$$

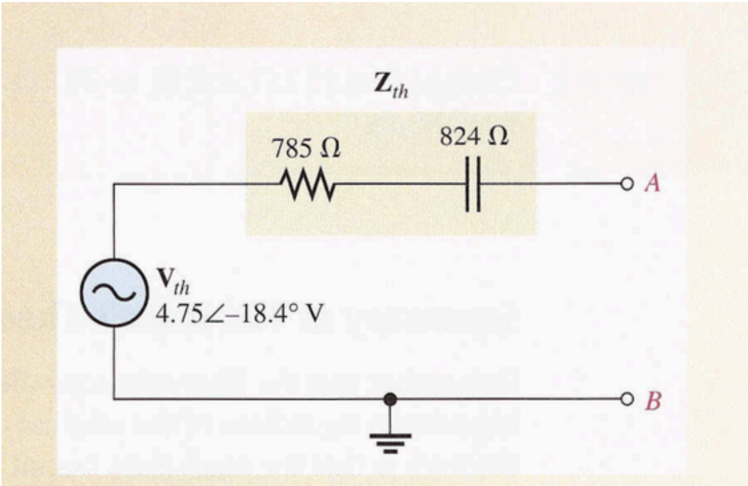
$$= 560\angle 0^\circ\ \Omega + 854\angle -74.7^\circ\ \Omega = 560\ \Omega + 225\ \Omega - j824\ \Omega$$

$$= 785\ \Omega - j824\ \Omega = \mathbf{1138\angle -46.4^\circ\ \Omega}$$

In rectangular form, the impedance is:

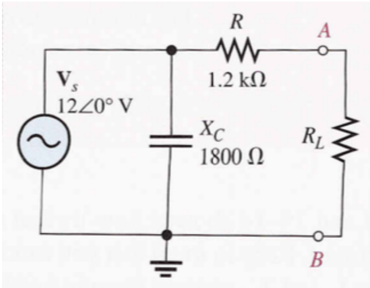
$$Z_{th} = 785\Omega - j824\Omega$$

The Thevenin equivalent circuit is as below:



Question # 6:

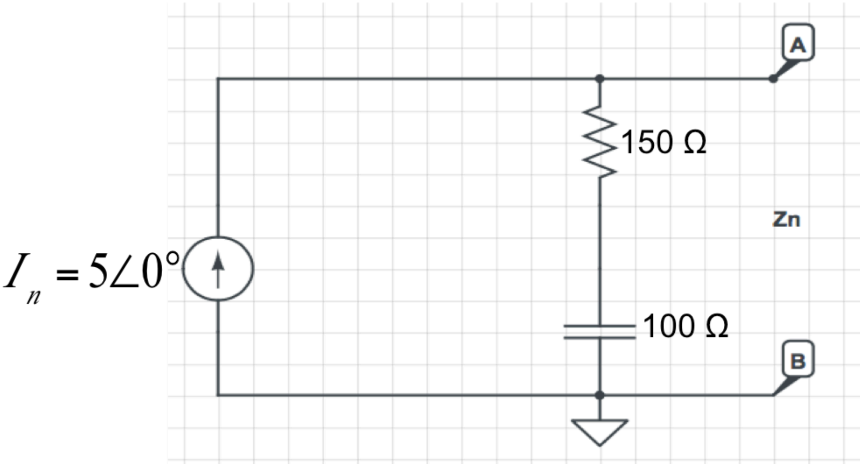
For a given circuit, $\boldsymbol{I_n} = 5\angle 0°\ mA$, and $\boldsymbol{Z_n} = 150\ \Omega + j100\Omega$. Draw the Norton equivalent circuit.



(Marks 04)

Solution:

The Norton equivalent circuit is as follows:



Good Luck