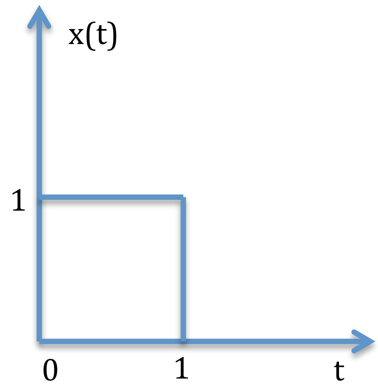
ISRA UNIVERSITY
Islamabad Campus

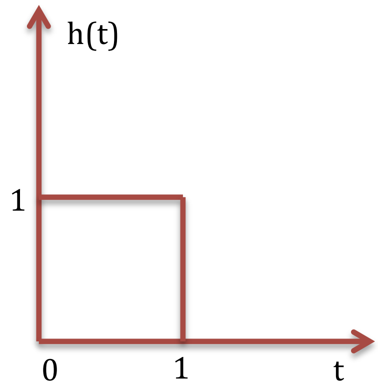
Program: BTECH (Electrical)
Semester – Spring 2018

ETSS-314 Signal & Systems

Assignment – 4 & 5 Solution **Due Date: 20/06/2018**
Marks: 50 **Handout Date: 29/05/2018**

Question # 1:
Graphically convolve the two signals shown below:





(Marks 10)

Solution:

Question # 2:
Find the Fourier Series Coefficients of the following signals:

1. $x[n] = sin(\omega_0 n), where\ \omega_0 = \frac{2\pi}{N}$
2. $x[n] = 1 + 3\cos\frac{2\pi n}{N}$

(Marks 07)

Solution:

1. $x[n] = sin(\omega_0 n), where\ \omega_0 = \frac{2\pi}{N}$

$$sin\left(\frac{2\pi n}{N}\right) = \frac{1}{2j}\left(e^{j\frac{2\pi n}{N}} - e^{-j\frac{2\pi n}{N}}\right)$$

The Fourier series coefficients are:

$$a_0 = 0, a_1 = \frac{1}{2j}, a_{-1} = -\frac{1}{2j}$$

3. $x[n] = 1 + 3\cos\frac{2\pi n}{N}$

$$x[n] = 1 + \frac{3}{2}\left(e^{j\frac{2\pi n}{N}} + e^{-j\frac{2\pi n}{N}}\right)$$

The Fourier series coefficients are:

$$a_0 = 1, a_2 = \frac{3}{2}, a_{-2} = \frac{3}{2}$$

Question # 3:

Determine system function of a system having impulse response of $h(t) = e^{-3t}u\ (t)$. If the input to this system is $x(t) = e^{-2t}u\ (t)$, find its output y(t) using Fourier transform.

(Hint: $e^{-at}u(t) = \frac{1}{a+j\omega}$)

(Marks 07)

Solution:

Let the output of the system by y (t). We know that:

$$Y(j\omega) = X(j\omega)H(j\omega)$$

Here:

$$H(j\omega) = \frac{1}{3 + j\omega}$$

And:

$$X(j\omega) = \frac{1}{2 + j\omega}$$

Therefore,

$$Y(j\omega) = \left[\frac{1}{2 + j\omega}\right]\left[\frac{1}{3 + j\omega}\right]$$

$$Y(j\omega) = \frac{1}{(2 + j\omega)(3 + j\omega)}$$

Using partial fractions expansion we get:

$$Y(j\omega) = \frac{1}{(2 + j\omega)(3 + j\omega)} = \frac{A}{(2 + j\omega)} + \frac{B}{(3 + j\omega)} \rightarrow eq1$$

Cross multiplication yields:

$$1 = A(3 + j\omega) + B(2 + j\omega)$$

Putting $j\omega = -3$, gives:

$$1 = A(3 + (-3)) + B(2 + (-3))$$

$$1 = A(3 - 3) + B(2 - 3)$$

$$1 = B(-1) \Rightarrow B = -1$$

$$1 = A(3 + j\omega) + B(2 + j\omega)$$

Putting $j\omega = -2$, gives:

$$1 = A(3 + (-2)) + B(2 + (-2))$$
$$1 = A(3 - 2) + B(2 - 2)$$
$$1 = A(1) \Longrightarrow A = 1$$

Putting values of A and B in eq(1) gives:

$$Y(j\omega) = \frac{1}{(2 + j\omega)} + \frac{-1}{(3 + j\omega)} = \frac{1}{(2 + j\omega)} - \frac{1}{(3 + j\omega)}$$

Taking the inverse Fourier transform, we obtain:

$$y(t) = e^{-2t}u(t) - e^{-3t}u(t)$$

<u>Question # 4:</u>

Consider a discrete-time LTI system with impulse response:

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Use the Fourier transforms to determine the response y [n] to the given input:

$$x[n] = \left(\frac{3}{4}\right)^n u[n]$$

(Marks 06)

<u>Solution:</u>

Let the output of the system by y [n]. We know that:

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

Here:

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}$$

And:

$$X(e^{j\omega}) = \frac{1}{1 - \frac{3}{4}e^{-j\omega}}$$

Therefore,

$$Y(e^{j\omega}) = \left[\frac{1}{1 - \frac{3}{4}e^{-j\omega}}\right]\left[\frac{1}{1 - \frac{1}{2}e^{-j\omega}}\right]$$
$$Y(e^{j\omega}) = \frac{1}{\left(1 - \frac{3}{4}e^{-j\omega}\right)\left(1 - \frac{1}{2}e^{-j\omega}\right)}$$

Using partial fractions expansion we get:

$$Y(e^{j\omega}) = \frac{1}{\left(1 - \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{3}{4}e^{-j\omega}\right)} = \frac{A}{\left(1 - \frac{1}{2}e^{-j\omega}\right)} + \frac{B}{\left(1 - \frac{3}{4}e^{-j\omega}\right)} \rightarrow eq1$$

Cross multiplication yields:

$$1 = A\left(1 - \frac{3}{4}e^{-j\omega}\right) + B\left(1 - \frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = 2$, gives:

$$1 = A\left(1 - \frac{3}{4}\times(2)\right) + B\left(1 - \frac{1}{2}\times(2)\right)$$

$$1 = A\left(\frac{2-3}{2}\right) + B(0)$$

$$1 = A\left(\frac{-1}{2}\right) \Longrightarrow A = -2$$

$$1 = A\left(1 - \frac{3}{4}e^{-j\omega}\right) + B\left(1 - \frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = \frac{4}{3}$, gives:

$$1 = A\left(1 - \frac{3}{4}\times\left(\frac{4}{3}\right)\right) + B\left(1 - \frac{1}{2}\times\left(\frac{4}{3}\right)\right)$$

$$1 = A(0) + B\left(\frac{3-2}{3}\right)$$

$$1 = B\left(\frac{1}{3}\right) \Longrightarrow B = 3$$

Putting values of A and B in eq(1) gives:

$$Y(e^{j\omega}) = \frac{-2}{\left(1 - \frac{1}{2}e^{-j\omega}\right)} + \frac{3}{\left(1 - \frac{3}{4}e^{-j\omega}\right)}$$

Taking the inverse Fourier transform, we obtain:

$$y[n] = 3\left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{1}{2}\right)^n u[n]$$

Question # 5:

Using partial fraction expansion and the fact that:

$$(a)^n u[n] \leftrightarrow \frac{1}{1-az^{-1}} \;\;,\;\; |z| > |a|$$

Find the inverse z-transform of:

$$X(z) = \frac{1-\frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})} \;,\;\; |z| > 2$$

(Marks 10)

Solution:

Using partial fraction expansion:

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})} = \frac{A}{(1-z^{-1})} + \frac{B}{(1+2z^{-1})} \rightarrow eq(1)$$

Cross multiplication gives:

$$1 - \frac{1}{3}z^{-1} = A(1+2z^{-1}) + B(1-z^{-1})$$

Putting $z^{-1} = 1$, gives:

$$1 - \frac{1}{3}(1) = A\big(1 + 2(1)\big) + B(1-1)$$

$$\frac{3-1}{3} = A(1+2) + B(0)$$

$$\frac{2}{3} = A(3) \Rightarrow A = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

Putting $z^{-1} = -\frac{1}{2}$, gives:

$$1 - \frac{1}{3}\left(-\frac{1}{2}\right) = A\left(1 + 2\left(-\frac{1}{2}\right)\right) + B\left(1 - \left(-\frac{1}{2}\right)\right)$$

$$1 + \frac{1}{6} = A(0) + B\left(1 + \frac{1}{2}\right)$$

$$\frac{6+1}{6} = B\left(\frac{2+1}{2}\right)$$

$$\frac{7}{6} = B\left(\frac{3}{2}\right) \Rightarrow A = \frac{7}{6} \times \frac{2}{3} = \frac{7}{9}$$

Putting values of A and B in eq(1) gives:

$$X(z) = \frac{\frac{2}{9}}{(1 - z^{-1})} + \frac{\frac{7}{9}}{(1 + 2z^{-1})}$$

Taking the inverse z-transform, we obtain:

$$x[n] = \frac{2}{9}u[n] + \frac{7}{9}(-2)^n u[n]$$

Question # 6:

Using the partial fraction expansion, determine the sequence x [n] that goes with the following z-transform:

$$X(z) = \frac{3z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

(Marks 10)

Solution:

Since x [n] is absolutely summable, the ROC must be $|z| > \frac{1}{2}$ in order to include the unit circle.

Using partial fraction expansion:

$$X(z) = \frac{3z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)} = \frac{A}{\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{B}{\left(1 + \frac{1}{4}z^{-1}\right)} \rightarrow eq(2)$$

Cross multiplication gives:

$$3z^{-1} = A\left(1 + \frac{1}{4}z^{-1}\right) + B\left(1 - \frac{1}{2}z^{-1}\right)$$

Putting $z^{-1} = -4$, gives:

$$3(-4) = A\left(1 + \frac{1}{4}(-4)\right) + B\left(1 - \frac{1}{2}(-4)\right)$$

$$-12 = A(0) + B(1 + 2)$$

$$-12 = B(3) \Rightarrow B = -\frac{12}{3} = -4$$

Putting $z^{-1} = 2$, gives:

$$3(2) = A\left(1 + \frac{1}{4}(2)\right) + B\left(1 - \frac{1}{2}(2)\right)$$

$$6 = A\left(1 + \frac{1}{2}\right) + B(0)$$

$$6 = A\left(\frac{2+1}{2}\right)$$

$$6 = A\left(\frac{3}{2}\right) \Rightarrow A = (6)\left(\frac{2}{3}\right) = 4$$

Putting values of A and B in eq (2) gives:

$$X(z) = \frac{4}{\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{-4}{\left(1 + \frac{1}{4}z^{-1}\right)} \Rightarrow \frac{4}{\left(1 - \frac{1}{2}z^{-1}\right)} - \frac{4}{\left(1 + \frac{1}{4}z^{-1}\right)}$$

Taking the inverse z-transform, we obtain:

$$x[n] = 4\left(\frac{1}{2}\right)^n u[n] - 4\left(-\frac{1}{4}\right)^n u[n]$$

Good Luck