

### Z-Transform Pairs

Sequence	Transform	ROC
1. $\delta[n]$	1	All $z$
2. $u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
3. $-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z  < 1$
4. $\delta[n - m]$	$z^{-m}$	All $z$ except 0 (if $m > 0$ ) or $\infty$ (if $m < 0$ )
5. $a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z  >  a $
6. $-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z  <  a $
7. $na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  >  a $
8. $-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  <  a $
9. $[\cos \omega_0 n]u[n]$	$\frac{1 - [\cos \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z  > 1$
10. $[\sin \omega_0 n]u[n]$	$\frac{[\sin \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z  > 1$
11. $[r^n \cos \omega_0 n]u[n]$	$\frac{1 - [r \cos \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z  > r$
12. $[r^n \sin \omega_0 n]u[n]$	$\frac{[r \sin \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z  > r$
13. $\begin{cases} a^n, & 0 \leq n \leq N - 1, \\ 0, & \text{otherwise} \end{cases}$	$\frac{1 - a^N z^{-N}}{1 - az^{-1}}$	$ z  > 0$

## Z-Transform Properties

**TABLE 10.1** PROPERTIES OF THE z-TRANSFORM

Section	Property	Signal	z-Transform	ROC
		$x[n]$	$X(z)$	$R$
		$x_1[n]$	$X_1(z)$	$R_1$
		$x_2[n]$	$X_2(z)$	$R_2$
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10.5.1	Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of $R_1$ and $R_2$
10.5.2	Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	$R$ , except for the possible addition or deletion of the origin
10.5.3	Scaling in the z-domain	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	$R$
		$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
		$a^n x[n]$	$X(a^{-1}z)$	Scaled version of $R$ (i.e., $aR$ = the set of points $\{ a z\}$ for $z$ in $R$ )
10.5.4	Time reversal	$x[-n]$	$X(z^{-1})$	Inverted $R$ (i.e., $R^{-1}$ = the set of points $z^{-1}$ , where $z$ is in $R$ )
10.5.5	Time expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer $r$	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$ , where $z$ is in $R$ )
10.5.6	Conjugation	$x^*[n]$	$X^*(z^*)$	$R$
10.5.7	Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of $R_1$ and $R_2$
10.5.7	First difference	$x[n] - x[n - 1]$	$(1 - z^{-1})X(z)$	At least the intersection of $R$ and $ z  > 0$
10.5.7	Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - z^{-1}}X(z)$	At least the intersection of $R$ and $ z  > 1$
10.5.8	Differentiation in the z-domain	$nx[n]$	$z \frac{dX(z)}{dz}$	$R$
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10.5.9	<b>Initial Value Theorem</b> If $x[n] = 0$ for $n < 0$ , then $x[0] = \lim_{z \rightarrow \infty} X(z)$			