24th SEPT, 18

LECTURE #2

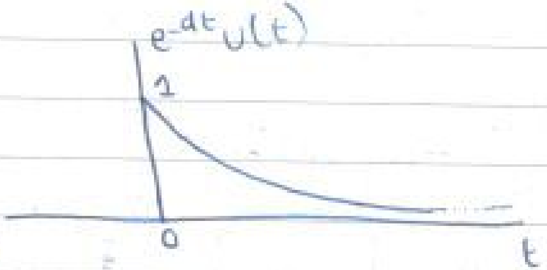
ENERGY & POWER :-

EXAMPLE #1 :-

a) Total Energy = ?

$$x(t) = e^{-\alpha t} u(t) \quad , \quad \alpha > 0$$

SOL :-



$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} |e^{-\alpha t} u(t)|^2 \, dt$$

$$= \int_{-\infty}^{0} 0 \, dt + \int_{0}^{\infty} (e^{-\alpha t})^2 \, dt \Rightarrow \int_{0}^{\infty} e^{-2\alpha t} \, dt$$

$$= \frac{-1}{2\alpha} \left[e^{-2\alpha t} \right]_0^{\infty} \Rightarrow \frac{-1}{2\alpha} \left[\overset{0}{\cancel{e^{-2\alpha(\infty)}}} - \overset{1}{\cancel{e^{0}}} \right]$$

$$E = \frac{-1}{2\alpha} (-1) \Rightarrow \frac{1}{2\alpha} \quad \text{(finite)}$$

Hence it is an Energy signal and power $P = 0$.

b) Calculate average power ?

$$x(t) = A_0 \sin \omega_0 t$$

<u>Sol</u>

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A_0^2 \sin^2 \omega_0 t \, dt$$

$$= \lim_{T\to\infty} \frac{A_0^2}{2T} \int_{-T}^{T} \left(\frac{1 - \cos 2\omega_0 t}{2}\right) dt$$

$$P = \lim_{T\to\infty} \frac{A_0^2}{4T}\left[\int_{-T}^{T} 1dt - \int_{-T}^{T} \cos 2w_0 t\, dt\right]$$

$$= \lim_{T\to\infty} \frac{A_0^2}{4T}\left[|t|_{-T}^{T} - \frac{1}{2w_0}\left|\sin 2w_0 t\right|_{-T}^{T}\right]$$

$$= \lim_{T\to\infty} \frac{A_0^2}{4T}\left[T-(-T)\right] - \lim_{T\to\infty} \frac{A^2}{8w_0 T}\left[\sin 2w_0 T - \sin 2w_0(-T)\right]$$

$$= \lim_{T\to\infty} \frac{A_0^2}{\cancel{4T}_{2}}\left[\cancel{2T}\right] - \underbrace{\lim_{T\to\infty} \frac{A^2}{8w_0 T}\left[\sin 2w_0 T - \sin(2w_0 T)\right]}_{0}$$

After applying limit sin terms does not exist

$$P = \frac{A_0^2}{2} \text{ ans.}$$

$$Rms = \sqrt{P} = \sqrt{\frac{A_0^2}{2}} \Rightarrow \frac{A_0^2}{\sqrt{2}}$$

EXAMPLE #2:-

Determine whether the following signals are energy signals or power signals:

~~a) $x[n] = \text{ramp}[n]$~~
~~Sol:~~

a) $x[n] = (1/3)^n u[n]$

Sol:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} \left|(1/3)^n u[n]\right|^2$$

$$= \sum_{n=0}^{\infty} \left|(1/3)^n\right|^2$$

$$E = \sum_{n=0}^{\infty} (1/9)^n$$

using Geometric Series formula : $S = \sum_{n=0}^{\infty} a^n \Rightarrow \frac{1}{1-a}$

~~Hence,~~

Hence, $E = \dfrac{1}{1-\frac{1}{9}}$

$$= \frac{1}{\frac{9-1}{9}} \Rightarrow \frac{9}{8} < \infty$$

Therefore it is an Energy signal.

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b) $x(n) = Au(n)$

Sol:

$$P = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |A_0 u(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |A_0|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} A_0^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left\{ A_0^2 + A_0^2 + \dots + A_0^2 \right\}$$

$n=1$ to $n=N \Rightarrow NA^2$

$$= \lim_{N\to\infty} \frac{A_0^2(1+N)}{(2N+1)}$$

$$= \lim_{N\to\infty} \frac{\cancel{N} A_0^2 \left(\frac{1}{N} + 1\right)}{\cancel{N}(2 + 1/N)}$$

After putting limit

$$= \frac{A_0^2\left(\cancel{1/\infty}^{\,0} + 1\right)}{2 + \cancel{1/\infty}_{\,0}} \Rightarrow P = \frac{A_0^2}{2} \text{ ans}$$

EXAMPLE # 3:-

Time period = ?

a) $x(t) = A_0 e^{j\omega_0 t}$

Sol:-

$$x(t) = A_0 e^{j\omega_0 t}$$

$$x(t+T) = A_0 e^{j\omega_0(t+T_0)} \rightarrow (1)$$

As $x(t) = x(t+T) = A_0 e^{j\omega_0 t}$

equ (1) becomes

$$\cancel{A_0} e^{j\omega_0 t} = \cancel{A_0} e^{j\omega_0(t+T_0)}$$

$$\cancel{e^{j\omega_0 t}} = \cancel{e^{j\omega_0 t}} \cdot e^{j\omega_0 T_0}$$

$$e^{j\omega_0 T_0} = 1$$

From euler's equation $e^{jx} = \cos x + j\sin x$ let say $x = 2\pi k$

$$e^{j2\pi k} = \underbrace{\cos \cancel{2}\pi k}_{1} + j\underbrace{\sin \cancel{2}\pi k}_{0}$$

$$e^{j2\pi k} = 1$$

$$e^{j\omega_0 T_0} = e^{j2\pi k}$$

$\therefore \omega_0 T_0 = 2\pi k$

$$T_0 = \frac{2\pi k}{\omega_0} \quad \text{if } k = 1$$

$$T_0 = \frac{2\pi}{\omega_0}$$

2) $x[n] = \cos\left[\frac{3\pi}{4} n\right]$

Sol:-

$$x[n] = \cos\left[\frac{3\pi}{4} n\right]$$

$$\omega_0 = \frac{3\pi}{4}$$

$$\frac{2\pi}{\omega_0} = \frac{2\cancel{\pi}}{3\cancel{\pi}/4} \Rightarrow \frac{8}{3} \text{ (Rational)}$$

$$\frac{N}{K} = \frac{2\pi}{\omega_0} \Rightarrow \frac{8}{3}$$

$$N = 8 \quad , k = 3$$

Fundamental Period $= N = 8$

EXAMPLE # 4:-

Calculate the fundamental period of composite signals.

a) $x(t) = \sin 6\pi t + \cos 5\pi t$

Sol:-

$$x(t) = \underbrace{\sin 6\pi t}_{x_1(t)} + \underbrace{\cos 5\pi t}_{x_2(t)}$$

STEP#1

$x_1(t) = \sin 6\pi t \quad , \quad x_2(t) = \cos 5\pi t$

$\omega_1 = 6\pi \quad , \quad \omega_2 = 5\pi$

$T_1 = \frac{2\pi}{\omega_1} \quad , \quad T_2 = \frac{2\pi}{\omega_2}$

$T_1 = \frac{\cancel{2}\cancel{\pi}}{_3\cancel{6}\cancel{\pi}} \Rightarrow \frac{1}{3} \quad , \quad T_2 = \frac{2\cancel{\pi}}{5\cancel{\pi}} \Rightarrow \frac{2}{5}$

STEP#2

$$\frac{T_1}{T_2} = \frac{1}{3} \Big/ \frac{2}{5} = \frac{1}{3} \times \frac{5}{2} \Rightarrow \frac{5}{6} \text{ (Rational)}$$

Step #3:

$$T_0 = LCM(T_1, T_2)$$

$$T_1 = \frac{1}{3}, \quad T_2 = \frac{2}{5}$$

$$T_0 = LCM\left(\frac{1}{3}, \frac{2}{5}\right)$$

$$T_0 = \frac{\text{LCM of numerator}}{\text{HCF of denominator}}$$

$$T_0 = \frac{LCM(1,2)}{HCF(3,5)} = \frac{2}{1} \Rightarrow 2 \text{ sec} \quad \underline{\text{Ans}}$$

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b) $x[n] = \sin\left[\frac{3\pi}{4}n\right] + \cos\left[\frac{5\pi}{7}n\right]$

Sol:

$$x[n] = \underbrace{\sin\left[\frac{3\pi}{4}n\right]}_{x_1[n]} + \underbrace{\cos\left[\frac{5\pi}{7}n\right]}_{x_2[n]}$$

Step 1:- $x_1[n] = \sin\left[\frac{3\pi}{4}n\right]$, $x_2[n] = \cos\left[\frac{5\pi}{7}n\right]$

$\omega_1 = \frac{3\pi}{4}$, $\omega_2 = \frac{5\pi}{7}$

$\frac{N_1}{k} = \frac{2\pi}{3\pi/4} = \frac{2\times4}{3}$, $\frac{N_2}{K} = \frac{2\pi}{5\pi/7} \Rightarrow \frac{2\times7}{5}$

$\frac{N_1}{k} = \frac{8}{3}$, $\frac{N_2}{K} = \frac{14}{5}$

$N_1 = 8$, $N_2 = 14$

Step #2:- $\frac{N_1}{N_2} = \frac{\cancel{8}^{\,4}}{\cancel{14}_{\,7}} \Rightarrow \frac{4}{7}$ (Rational)

Step #3:- $N = LCM(8, 14)$

$N = 56$ Ans

EXAMPLE #5:-

Find the even and odd components of the following signals:-

a) $x(t) = \cos(t) + \sin(t) + \cos(t)\sin(t)$

Sol:-

$$x(t) = \cos(t) + \sin(t) + \cos(t)\sin(t) \rightarrow (1)$$

$$t \rightarrow -t$$

$$x(-t) = ?$$

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t)\sin(-t) \rightarrow (2)$$

$\therefore \cos(-\theta) \Rightarrow \cos\theta$ & $\sin(-\theta) \Rightarrow -\sin\theta$

then eqn (2) becomes

$$x(-t) = \cos t - \sin t - \cos t \sin t$$

For even we should have $x_1(t) = x_1(-t)$

$$x_e(t) = \cos t$$

$$x_o(t) = +\sin t + \cos t \sin t$$

OR by using formula:-

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$= \frac{1}{2}\left[\left(\cos(t) + \cancel{\sin(t)} + \cancel{\cos(t)\sin(t)}\right) + \left(\cos t \cancel{-\sin t} \cancel{-\cos t \sin t}\right)\right]$$

$$x_e(t) = \frac{1}{\cancel{2}}\left[\cancel{2}\cos(t)\right] \Rightarrow \cos(t)$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

$$= \frac{1}{2}\left[\{\cos(t) + \sin(t) + \cos(t)\sin(t)\} - \{\cos(t) - \sin(t) - \cos(t)\sin(t)\}\right]$$

$$x_o(t) = \frac{1}{2}\left[\cancel{\cos(t)} + \sin(t) + \cos(t)\sin(t)\right.$$
$$\left. - \cancel{\cos(t)} + \sin(t) + \cos(t)\sin(t)\right]$$
$$= \frac{1}{\cancel{2}}\left[\cancel{2}\sin(t)\right] + \frac{1}{\cancel{2}}\left[\cancel{2}\cos(t)\sin(t)\right]$$
$$x_o(t) = \sin(t) + \cos(t)\sin(t)$$

2) $x[n] = \{-4-5j, \underset{\uparrow}{1+2j}, 4\}$

Sol:-

$$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\}$$
$$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\}$$
$$x[-n] = \{4, \underset{\uparrow}{1+2j}, -4-5j\}$$
$$x_e[n] = \frac{1}{2}\left[\{-4-5j, \underset{\uparrow}{1+2j}, 4\} + \{4, \underset{\uparrow}{1+2j}, -4-5j\}\right]$$
$$= \frac{1}{2}\left[(-\cancel{4}-5j+\cancel{4}), \underset{\uparrow}{(1+2j+1+2j)}, (\cancel{4}+(-\cancel{4}-5j))\right]$$
$$= \frac{1}{2}[-5j, 2+4j, -5j]$$
$$x_e[n] = \left\{\frac{-5}{2}j, \frac{\cancel{2}(1+2j)}{\cancel{2}}, -\frac{5}{2}j\right\} \Rightarrow \left\{\frac{-5}{2}j, \underset{\uparrow}{1+2j}, -\frac{5}{2}j\right\}$$
$$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\} = \frac{1}{2}\left[\{-4-5j, \underset{\uparrow}{1+2j}, 4\} - \{4, \underset{\uparrow}{1+2j}, -4-5j\}\right]$$
$$= \frac{1}{2}\{(-4-5j-4), (\cancel{1}+\cancel{2j}-\cancel{1}-\cancel{2j}), (4+4+5j)\}$$
$$= \frac{1}{2}[(-8-5j), (0), (8+5j)]$$

$$x_0[n] = \frac{-8-5j}{2}, 0, \frac{8+5j}{2}$$