Signal & Systems

Lecture # 3

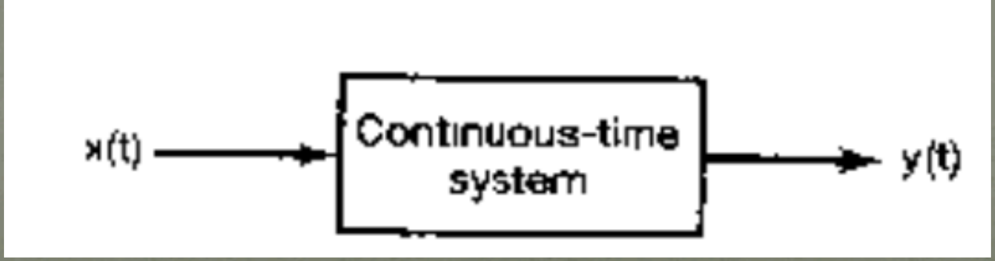
Continuous & Discrete Systems

8th October 18

Fundamentals of Systems

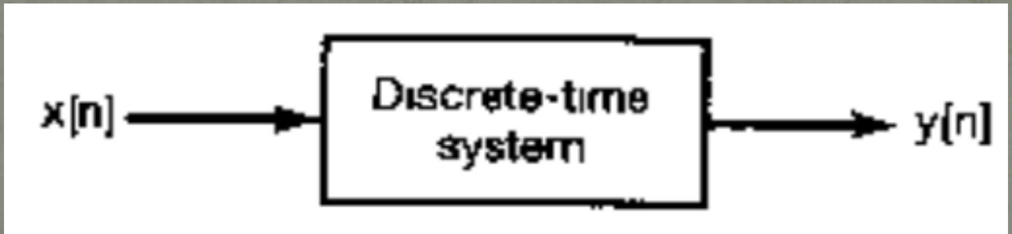
Systems

- A system in the broadcast sense are an interconnection of components, devices or subsystems.
- A system can be viewed as a process in which input signals are transformed by the system or cause the system to respond in some way resulting in other signals as output.
- A continuous time system is a system in which continuous time input signals are applied and result in continuous time output signals. The input-output relation is represented by the following notation: $x(t) \rightarrow y(t)$.



Systems (cont.)

- Similarly a discrete time system is a system that transforms discrete time inputs into discrete time outputs and represented symbolically as: x[n]➔y[n].



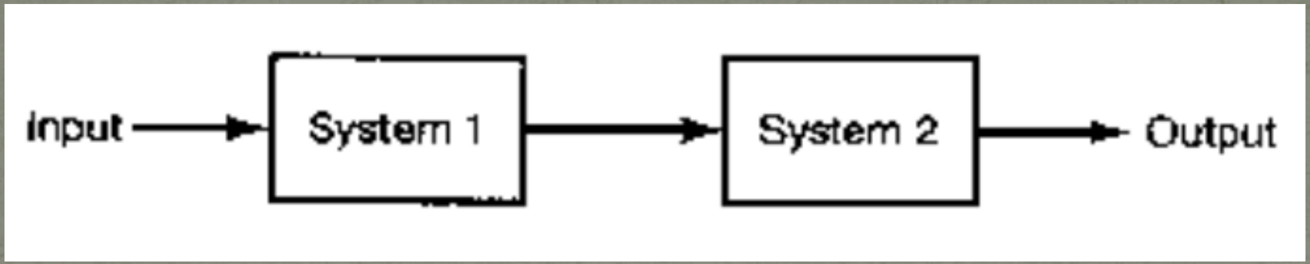
Interconnection of Systems

Interconnection

- Many real systems are built as interconnection of several subsystems.
- For example an audio system which involves the interconnection of a radio receiver, compact disc player or tapo deck with an amplifier and one or more speakers.
- By viewing a system as an interconnection of its components we can use our understanding of the component systems and of how they are interconnected in order to analyze the operation and behavior of the overall system.

Series Interconnection

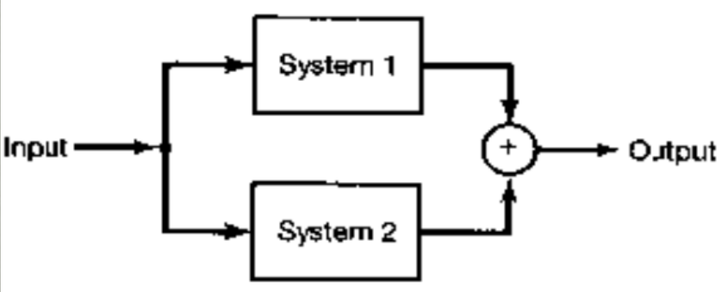
- A series or cascade interconnection of two systems shown below is referred to as a block diagram.



- An example of a series interconnection is a radio receiver followed by an amplifier.
- Similarly one can define a series interconnection of three or more systems.

Parallel Interconnection

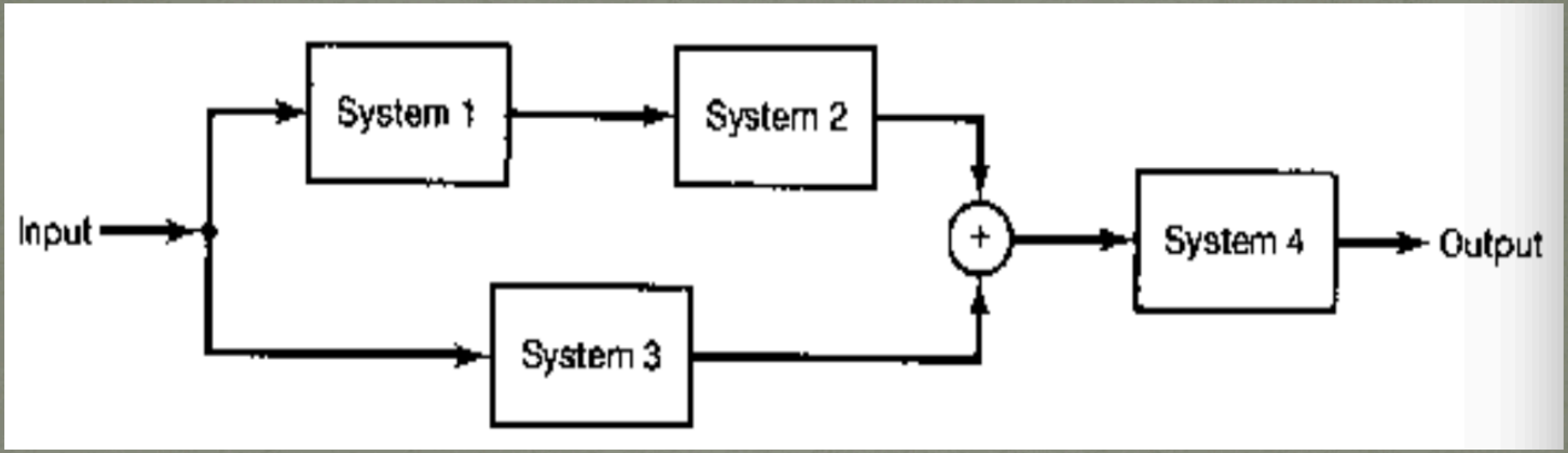
- A parallel interconnection of two systems is shown below:



- Here the same input signal is applied to Systems 1 and 2.
- The symbol "$\oplus$" denotes addition, so that the output of the parallel interconnection is the cum of the outputs of systems 1 and 2.
- We can define parallel interconnections of more than two systems, and we can combine both cascade and parallel interconnections to obtain more complicated interconnections.

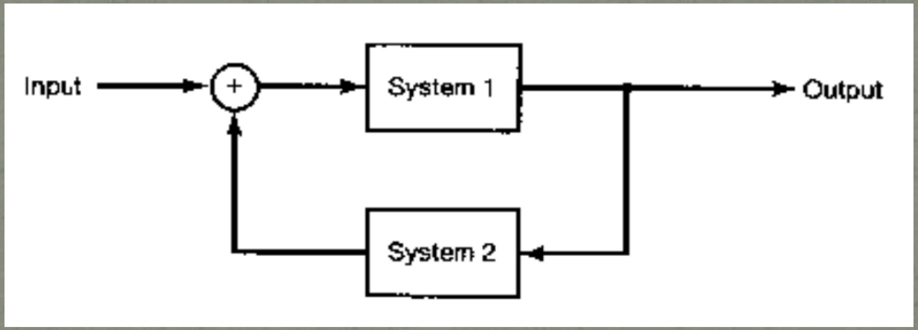
Parallel Interconnection (cont.)

- An example of such an interconnection is given below



Feedback Interconnection

- Feedback interconnection is shown below:



- Here the output of system 1 is the input to system 2 while the output of system 2 is fed back and added to the external input to produce the actual input to system 1.
- Feedback systems arise in a wide variety of applications.
- For example a cruise control system on an automobile senses the vehicle's velocity and adjusts the fuel flow in order to keep the speed at the desired level.

System Properties

Basic System Properties

- The basic system properties are as follows:
  - Static & Dynamic Systems
  - Causal & Non-Causal Systems
  - Time-Invariant & Time-Variant Systems
  - Linear & Non-Linear Systems
  - Invertible & Non-Invertible Systems
  - Stable & Unstable Systems

Static & Dynamic System

- Static system also known as Memory-less system.
- A system is said to be memory-less or static if its output for each value of the independent variable is dependent only on the input signal at that same time.
- For example:
  - $y(t) = 5x(t)$, y(t) is the output signal corresponding to the input signal x(t) is memory-less.
  - The system specified by the relationship below is memory-less as the value of y[n] at any particular time n_o depends only on the value of x[n] at that time.

$$y[n] = \left(2x[n] - x^2[n]\right)^2$$

Static & Dynamic System (cont.)

- The concept of memory in a system corresponds to the presence of a mechanism in the system that retains or stores information about input values at times other than the current time.
- Note:
  - The system will be dynamic (with memory) when there is time scaling.
  - The system will be dynamic (with memory) when there is time shifting.
  - Integration based systems will be dynamic as well.

Example #1

- Tell whether the following systems are memory-less or with memory system:
  - (1): $y(t) = x(2t)$
  - (2): $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$
  - (3): $y[n] = x[-n]$
  - (4): $y[n] = x[n+1]$

Causal & Non-Causal System

- A system is causal if the output at any time depends only on values of the input at the present time and in the past.
- Such a system is often referred to as being non-anticipative, as the system output does not anticipate future values of the input.
- Examples:
  - System with description $y(t) = x(t-1) + x(t)$ is clearly causal, as output $y(t)$ depends only on values of present and past input values.
  - $y[n] = x[n+1] + x[n]$ is not causal, because $x[n+1]$ is a future sample.
- Note: All memory-less systems are causal, since the output responds only to the current value of the input.

Causal & Non-Causal System (cont.)

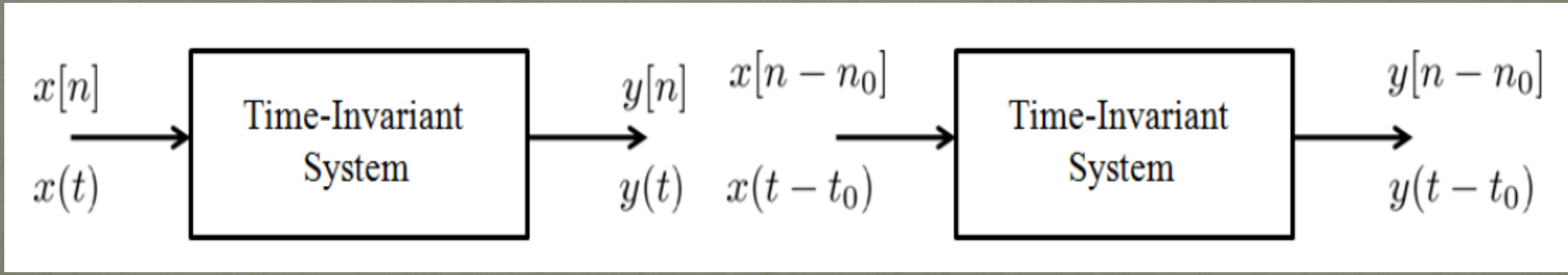
- There is an Anti-Causal system which depends only on the future values of input.
- All anti-causal systems are always non-causal but the reverse of this statement is not true.

Example #2

- Tell whether the following systems are causal or non-causal:
  - (1): $y(t) = x(3t)$
  - (2): $y(t) = \begin{cases} x(3t) & t < 0 \\ x(t-1) & t \geq 0 \end{cases}$
  - (3): $y[n] = x[n] + x[n-1]$

Time-Invariant & Time-Variant Systems

- A system is time invariant if the behavior and characteristics of the system are fixed over time.
- A system is time-invariant if a time-shift of the input signal results in the same time-shift of the output signal. That is if: x (t) ➔ y(t), then the system is time invariant if: x ($t-t_o$) ➔ y($t-t_o$) for any to belonging to R.
- Illustration of a time-invariant system:



Example #3

- Tell whether the following systems are time variant or time-invariant:
  - (1): $y(t) = x(\cos t)$
  - (2): $y[n] = x[2n]$

Linear & Non-Linear Systems

- A system is said to be linear if, for any two input signals, their linear combination yields as output the same linear combination of the corresponding output signals.
- A linear system in continuous time or discrete time is a system that possess the important property of superposition.
- A system is linear if it is additive and scalable. That is:

$$ax_1(t)+bx_2(t)\rightarrow ay_1(t)+by_2(t)$$

$$for \quad all \quad a,b\in C$$

$$ax_1[n]+bx_2[n]\rightarrow ay_1[n]+by_2[n]$$

Linear & Non-Linear Systems (cont.)

- Properties of Linearity:
  - System linearity is independent of time scaling.
  - System linearity is independent of coefficient used in system relationship.
  - If any added/subtracted term other than i/p and o/p is available in the system relationship then the system will be nonlinear.
  - If output is summation of time shifted terms of input, then the system will be linear.
  - Integral and Differential operators are linear operators.
  - Even and Odd operators are linear operators.

Linear & Non-Linear Systems (cont.)

- Real, Imaginary and Conjugate operators are nonlinear operators.
- Trigonometric, Inverse Trigonometric, Logarithmic, Exponential, Roots, Powers, sinc…….
- For zero i/p , o/p is also equal to zero.
- Split systems are linear systems.

Example #4

- Tell whether the following systems are linear or non-linear:
  - (1): $y(t) = e^3 x(t)$
  - (2): $y[n] = x[2n]$

Invertible & Non-Invertible Systems

- A system is said to be invertible if distinct inputs lead to distinct outputs.
- In other words, a system is invertible if there exists an one-to-one mapping from the set of input signals to the set of output signals.
- For example in systems for encoding used in a wide variety of communications applications. In such a system a signal that we wish to transmit is first applied as the input to a system known as an encoder.
- There are many reasons to do this, ranging from the desire to encrypt the original message for secure or private communication to the objective of providing some redundancy in the signal.

Invertible & Non-Invertible Systems (cont.)

- So that if any error occur in transmission can be detected and possibly corrected. For lossless coding the input to the encoder must be exactly recoverable from the output, i.e., the encoder must be invertible.
- One-to-one Mapping:
  - For each and every input value the output value will be unique.
- Many-to-one Mapping:
  - Many inputs have same outputs.
- For an Invertible system, there should be on to one mapping b/w input and output at each and every distinct of time. Otherwise the system will be non-invertible.

Example #5

- Tell whether the following system is invertible or non-invertible:

$$y(t) = \sin t \cdot x(t)$$

Stable & Un-Stable Systems

- A stable system is a one in which small inputs lead to predictable responses that do not diverge i.e., are bounded.
- For example: Consider an ideal mechanical spring. If we consider tension in the spring as a function of time as the input signal and elongation as a function of time to be the output signal, it would appear intuitively that the system is stable. A small tension leads only to a finite elongation.
- To describe a stable system, we first need to define the notion of BIBO stability, i.e., Bounded Input-Bounded Output Stability.

Stable & Un-Stable Systems (cont.)

- A system is said to be BIBO stable if for any bounded input signal, the output signal is bounded.
  - If $|x(t)| \leq B$ for some $B < \infty$, then $|y(t)| < \infty$.
- It is not necessary for the input and output signal to have the same independent variable for this property to make sense.
- It is valid for continuous time, discrete time and hybrid systems.

Stable & Un-Stable Systems (cont.)

- Bounded signal examples:
  - D.C. value: we will have finite amplitude from -∞ to ∞, i.e., $y(t)=6$.
  - $\sin(t)$ as amplitude varies from -1 to 1.
  - $\cos(t)$ as amplitude varies from -1 to 1.
  - $u(t)$ as it's amplitude will be 0 or 1.

Example #6

- Tell whether the following systems are stable or not:
  - (1): $y(t) = \dfrac{x(t)}{t}$
  - (2): $y[n] = \cos n \cdot x[n]$

The End