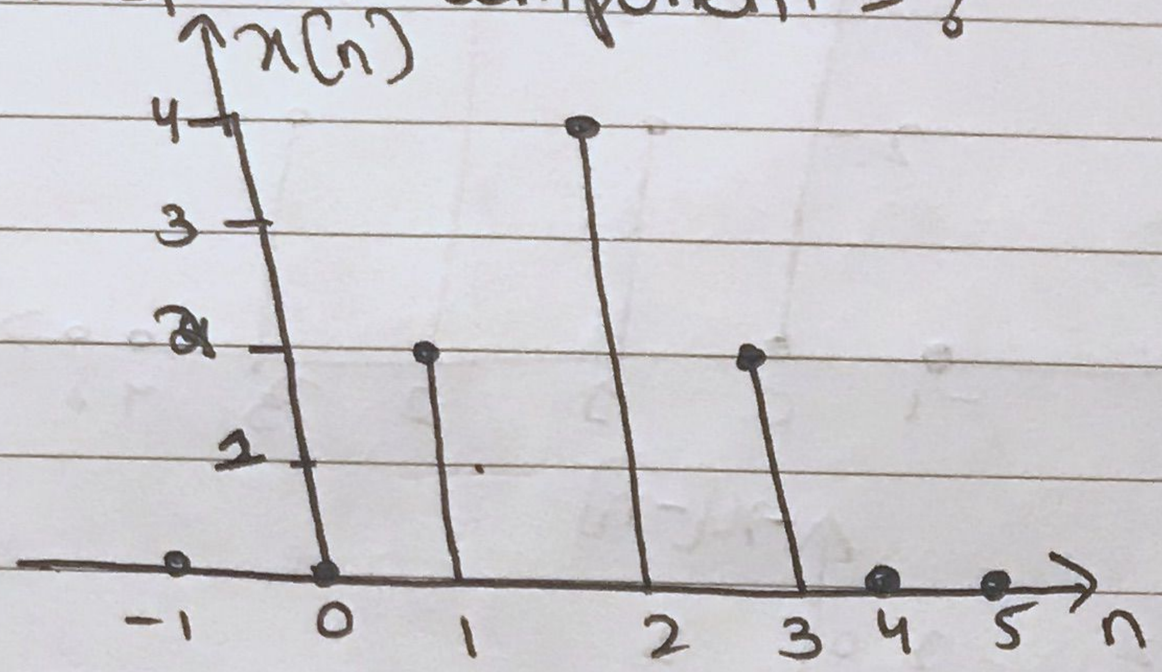
:- ASSIGNMENT #2 :-

Q#1 :-

Even & odd component = ?

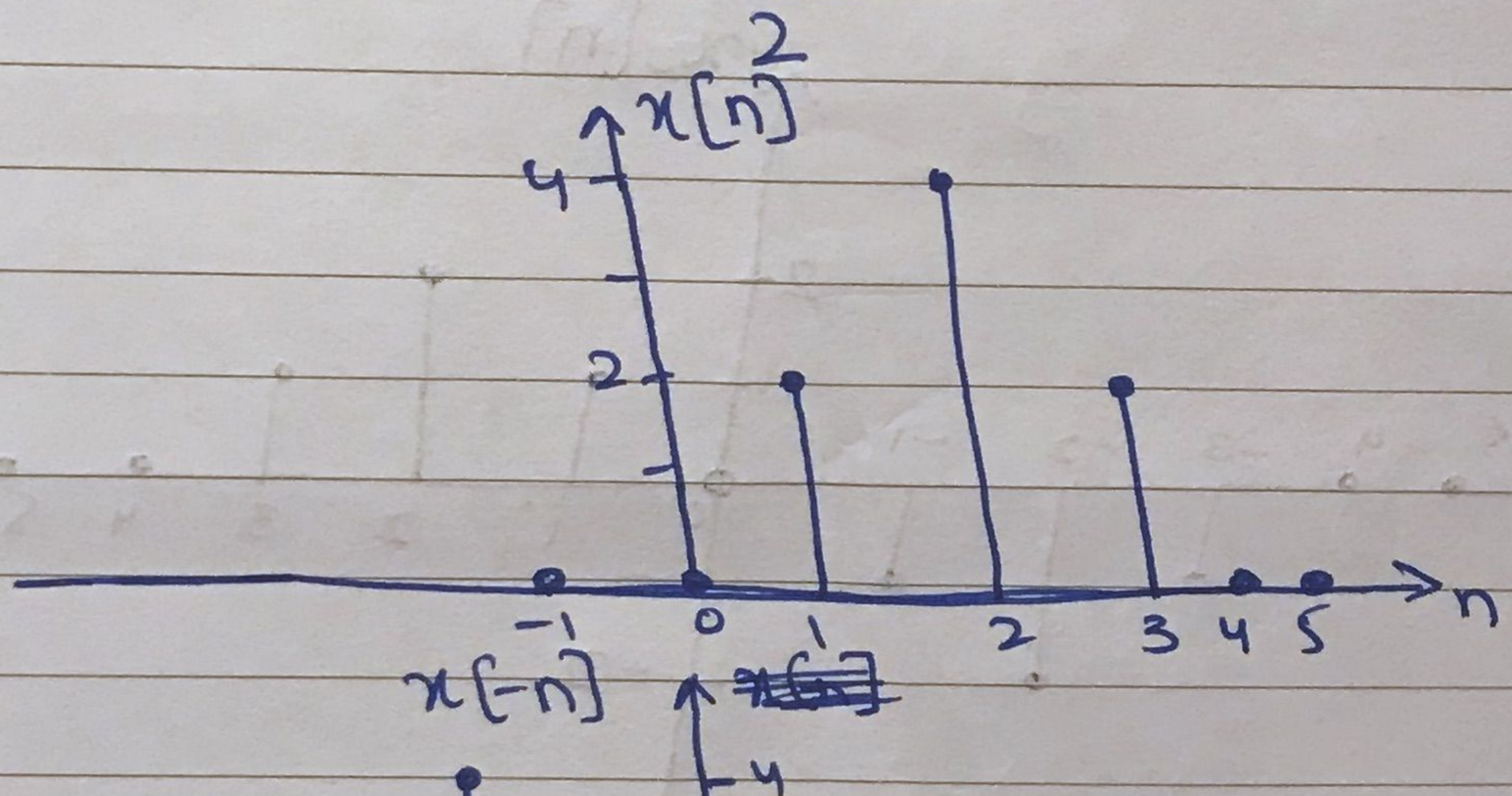


SOL :-

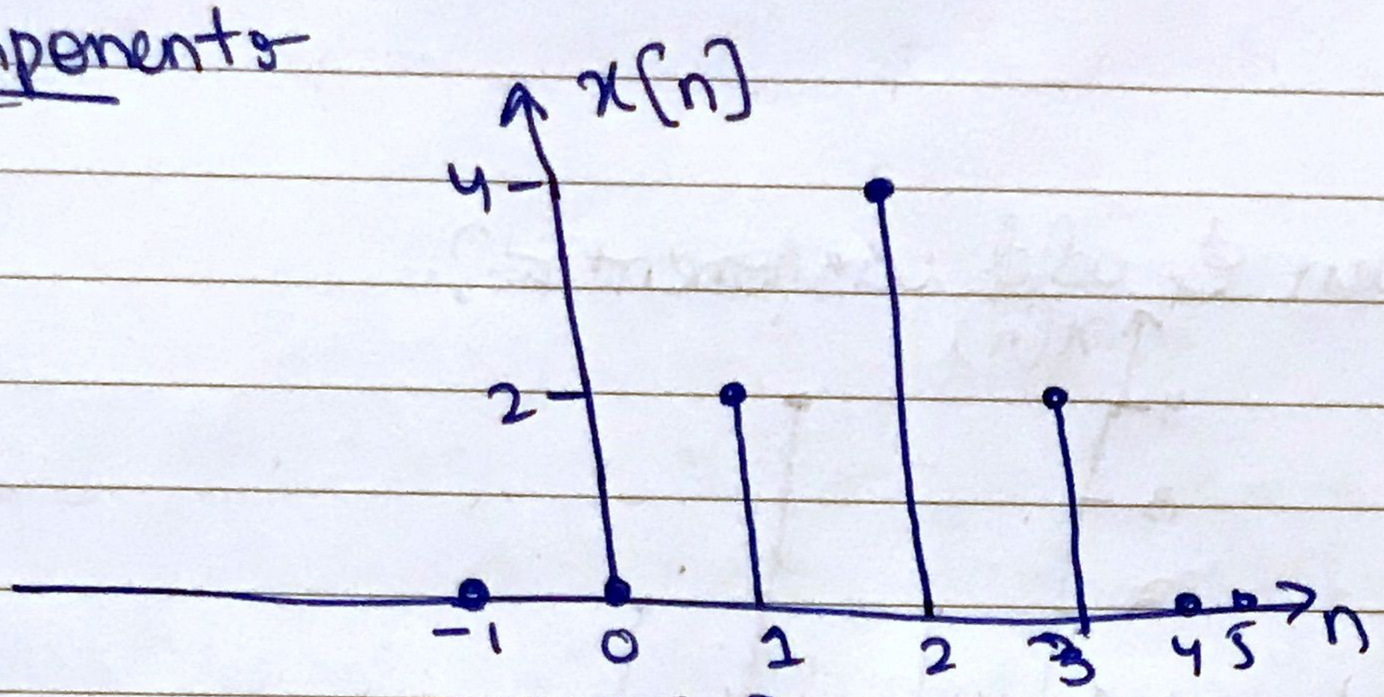
Even Component :-

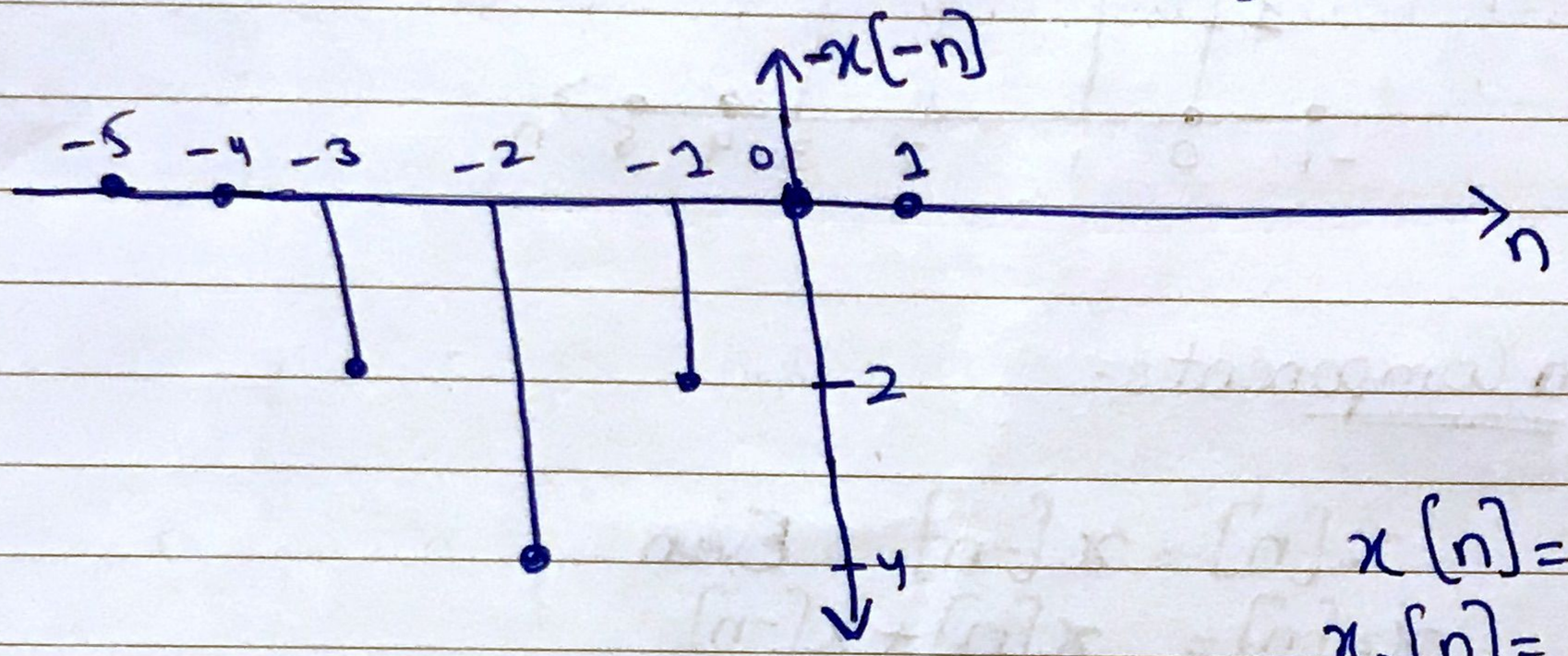
$$x[n] = x[-n] \quad \text{Even}$$

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$



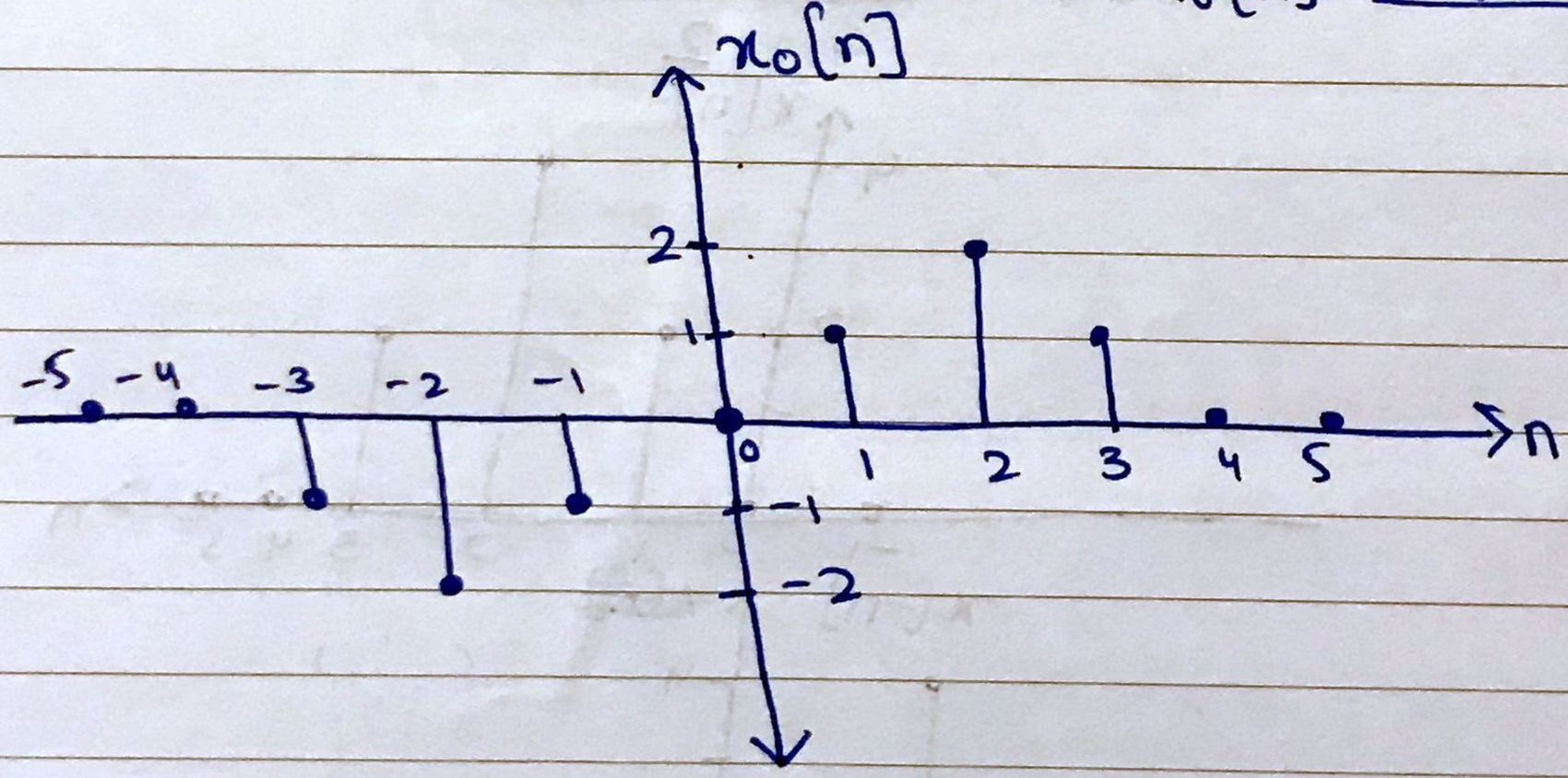
Odd Components:





$$x[n] = -x[-n] \quad \text{odd}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$



Q#2: Periodic or Aperiodic=?

$$x[n] = \cos^2 \frac{\pi}{8} n$$

Sol:

For a signal to be periodic $\Rightarrow x[n] = x[n \pm mN]$

$$x[n] = \cos^2 \frac{\pi}{8} n = \frac{1}{2} + \frac{1}{2} \cos \frac{\pi}{4} n$$

$$\Downarrow$$

$$\because \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

where $x[n] = \frac{1}{2} + \frac{1}{2}\cos\frac{\pi}{4}n$ composite signal

$$x_1[n] = \frac{1}{2} \Rightarrow \frac{1}{2}(1)^n \quad , \quad x_2[n] = \frac{1}{2}\cos\frac{\pi}{4}n$$

STEP #1 Find fundamental period N of $x_1[n]$ and $x_2[n]$.

$$N_1 = 1 \quad , \quad \frac{N_2}{m} = \frac{2\pi}{\omega_2} \quad \therefore \omega_2 = \frac{\pi}{4}$$

$$\frac{N_2}{m} = \frac{2\pi}{\pi/4} = \frac{2\cancel{\pi}}{\cancel{\pi}} \times 4 \Rightarrow \frac{8}{1}$$

$$N_2 \Rightarrow 8$$

STEP #2 Check rationality i-e $\frac{N_1}{N_2}$

$\frac{N_1}{N_2} = \frac{1}{8}$ rational. Hence $x[n]$ is periodic.

STEP #3 Check fundamental period of $x[n]$.

$$N_0 = LCM(N_1, N_2) = LCM(1, 8)$$

$$N_0 \Rightarrow 8.$$

Thus, $x[n]$ is periodic with fundamental period $N_0 = 8$.

Q#3: Energy / Power / NENP signal ?

$$x(t) = t\,u(t)$$

Sol:

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T\to\infty} \int_{-T/2}^{T/2} |t\,u(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T/2} t^2 dt = \lim_{T\to\infty} \left[\frac{t^3}{3}\right]_0^{T/2}$$

$$= \lim_{T\to\infty} \left[\frac{(T/2)^3}{3}\right] \Rightarrow \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} t^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{t^3}{3}\right]_0^{T/2} = \lim_{T\to\infty} \frac{1}{T} \frac{(T/2)^3}{3} \Rightarrow \infty$$

Thus, $x(t)$ is neither an energy nor a power signal i-e:, NENP.

Q#4:-

a) Linear / Non-linear
b) Time variant / Time invariant
c) With memory / Without memory.

$$y(t) = t\,x(t-2)$$

Sol:-

a) Linear / Non-linear:-

$$y(t) = t\,x(t-2)$$

Law of Additivity:-

$$x_1(t) = t\,x_1(t-2)$$
$$x_2(t) = t\,x_2(t-2)$$

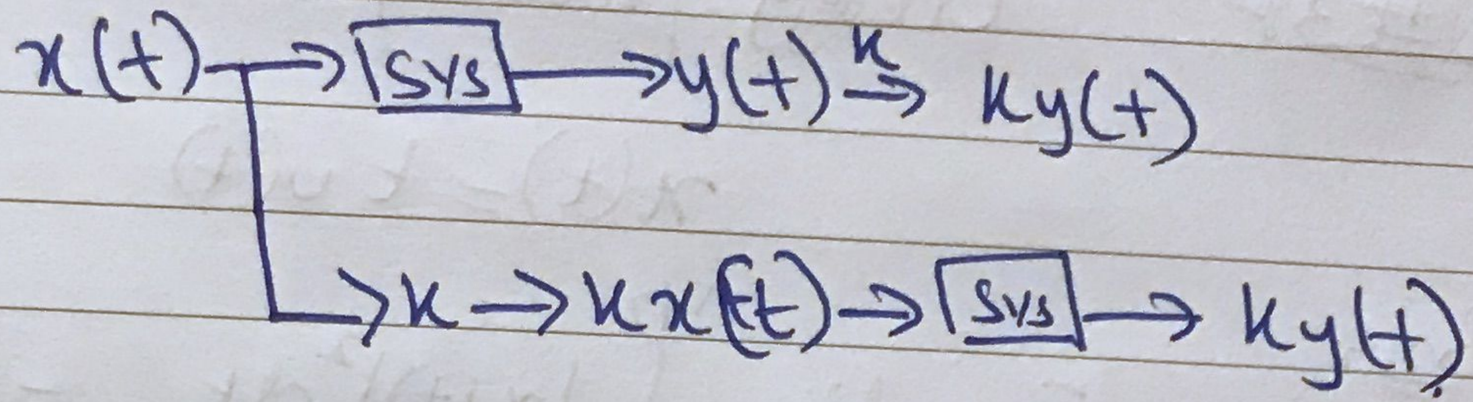
$x(t) \rightarrow$ [SYS] $\rightarrow y(t) = t\,x(t-2)$

$$y_3(t) = y_1(t) + y_2(t) = t\,x_1(t-2) + t\,x_2(t-2)$$

$x_1(t) + x_2(t) \rightarrow$ [SYS] $\rightarrow t\,x_1(t-2) + t\,x_2(t-2) = y'(t)$

$y_3(t) = y'(t)$, Hence law of Addition satisfied

Law of Homogenity



~~x(t)~~ $y(t) = t\,x(t-2)$

$k\,y(t) = k\,t\,x(t-2) \Rightarrow y'(t)$

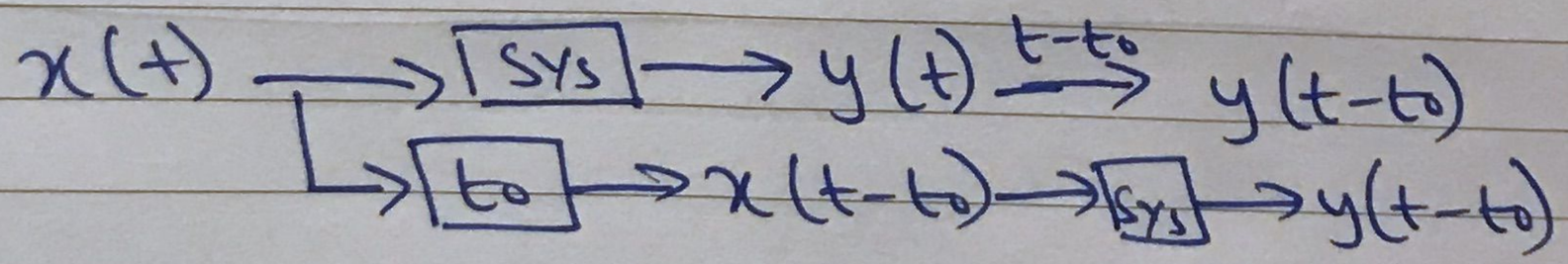
$x(t) \xrightarrow{k} kx(t) \rightarrow$ [Sys] $\rightarrow ky(t) = k\,t\,x(t-2) \Rightarrow y''(t)$

$y'(t) = y''(t)$ Hence satisfied.

As ~~both~~ both laws are satisfied then law of Superposition is ~~e~~ satisfied. Then the system ~~y~~ $y(t) = t\,x(t-2)$ is Linear.

$\longleftrightarrow$

b) Time variant | Time Invariant ~~s~~



$x(t) \rightarrow sys \rightarrow y(t) = t\,x(t-2) \rightarrow t = t - t_o \rightarrow y(t-t_o)$

$= (t-t_o)\,x(t-t_o-2)$

$x(t) \xrightarrow{t_o} x(t-t_o) \rightarrow sys \rightarrow y(t) = t\,x[(t-t_o)-2]$

Hence both the outputs are not same.
Then the system is Time variant.

c) Memoryless / With memory

$$y(t) = t\,x(t-2)$$

let's try different values of t

$\therefore t = -1$

$$y(-1) = -1\,x(-1-2) = -x(-3)$$

$\therefore t = 2$

$$y(2) = 2\,x(2-2) \Rightarrow 2x(0)$$

Hence present output is dependent on the past input, then the system is Memory system (with Memory).