22ND, OCT, 18

LECTURE #5
"PROPERTIES OF LTI"

EXAMPLE #1:-

$$y(t) = x(t - t_0)$$

Sol:-

$t_0 > 0$, a system is delay
$t_0 < 0$, a system is advance.

if $t_0 > 0$, then the o/p at time t equals the value of the input at the earlier time $t - t_0$.
if $t_0 = 0$ the system is the identity system and thus memoryless.
For any other value of t_0, this system has memory.
⇒ if ~~by~~ we take input equal to $\delta(t)$, we can obtaine the impulse response, i-e,

$$h(t) = \delta(t - t_0)$$

Therefore

$$\cancel{x(t)}\ x(t - t_0) = x(t) * \delta(t - t_0)$$

⇒ that is the convolution of a signal with a shifted impulse simply shifts the signal.

⇒ to recover the input from the output i-e to invert the system, all that is required is to shift the output back
⇒ The system with this compensating time shift is then the inverse system.
⇒ that is, if we take.

$$h_1(t) = \delta(t + t_0)$$

then

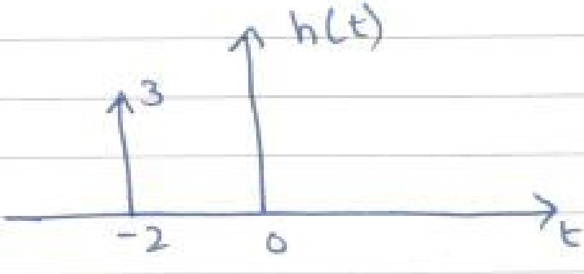
$$h(t) * h_1(t) = \delta(t - t_0) * \delta(t + t_0) = \delta(t).$$

EXAMPLE #2:

1) $h(t) = 3\delta(t+2)$

Causal or not = ?

Sol:



here $h(t) \neq 0$, $t < 0$, thus the system is non-causal.

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2) $h(t) = e^{-(t+1)} . u(t)$

Sol:

$$u(t) = \begin{cases} 0 & , t < 0 \\ 1 & , t \geq 0 \end{cases}$$

$$h(t) = \begin{cases} 0 & , t < 0 \\ e^{-(t+1)} & , t \geq 0 \end{cases}$$

Hence, the system is causal.

Example #3:

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

Sol:

It can also be expressed in the form,

$$y[n] = x[n] + \frac{1}{2}y[n-1]$$

Suppose that we impose the condition of initial rest and consider the input,

$$x[n] = k\,\delta[n]$$

In this case since $x[n]=0$ for $n \leq -1$, the condition of initial rest implies that $y[n]=0$ for $n \leq -1$, so that we have as an initial condition $y[-1]=0$.

Starting from this initial condition, we can solve for successive values of $y[n]$ for $n \geq 0$, as follows:

$$y[0] = x[0] + \frac{1}{2}y[-1] = k$$

$$y[1] = x[1] + \frac{1}{2}y[0] = \frac{1}{2}k$$

$$y[2] = x[2] + \frac{1}{2}y[1] = \left(\frac{1}{2}\right)^2 k$$

$$\vdots$$

$$y[n] = x[n] + \frac{1}{2}\{y[n-1] = \left(\frac{1}{2}\right)^n k$$

Setting $k=1$ we see that the impulse response for the system considered in this example is,

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$