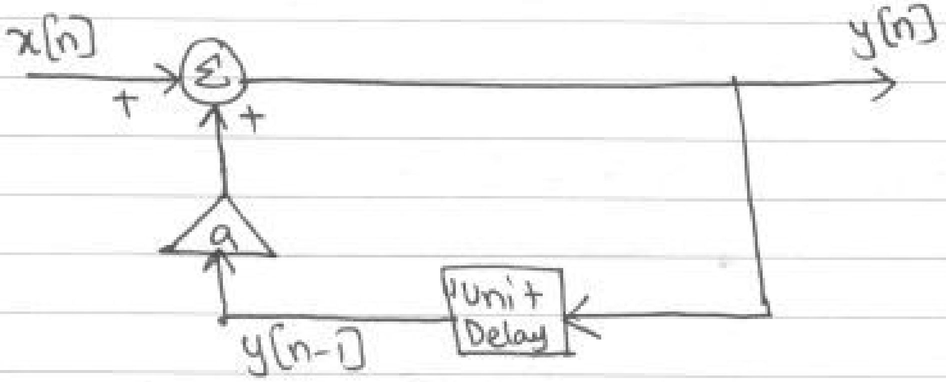
29-10-18

LECTURE # 6:

EXAMPLE # 1:



Sol:

The output of the unit delay element is $y[n-1]$.

$$y[n] = a\,y[n-1] + x[n]$$

or

$$y[n] - a\,y[n-1] = x[n]$$ $\Rightarrow$ First order linear Difference equation

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EXAMPLE # 2:

$$y[n] - \frac{1}{2} y[n-2] = 2x[n] - x[n-2]$$

$$h[n] = ?$$

Sol:

$$h[n] = \frac{1}{2} h[n-2] + 2\delta[n] - \delta[n-2]$$

Since the system is causal, $h[-2] = h[-1] = 0$. Then.

$$h[0] = \frac{1}{2} h[-2] + 2\delta[0] - \delta[n-2] = 2\delta[0] = 2$$

$$h[1] = \frac{1}{2} h[-1] + 2\delta[1] - \delta[-1] = 0$$

$$h[2] = \frac{1}{2}h[0] + 2\delta[2] - \delta[0] = \frac{1}{2}(2) - 1 = 0$$

$$h[3] = \frac{1}{2}h[1] + 2\delta[3] - \delta[1] = 0$$

$$\vdots$$

Hence, $h[n] = 2\delta[n]$

EXAMPLE #3:

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau)\, d\tau$$

Sol:-

a)

$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau) d\tau$$

$$= e^{-(t-\tau)} \Big|_{\tau=0} \Rightarrow e^{-t} \qquad t > 0$$

thus $\qquad h(t) = e^{-t} u(t)$

b) let $x(t) = e^{st}$. Then

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} e^{s\tau} d\tau$$

$$= \int_{-\infty}^{t} e^{-t} e^{\tau} e^{s\tau} d\tau = e^{-t} \int_{-\infty}^{t} e^{\tau(s+1)} d\tau$$

$$= e^{-t} \left[\frac{1}{s+1} e^{\tau(s+1)} \right]_{-\infty}^{t}$$

$$= e^{-t} \left[\frac{e^{t(s+1)}}{s+1} - \cancel{\frac{e^{-\infty}}{s+1}}^{\,0} \right]$$

$$= \frac{e^{-t} \cdot e^{t(s+1)}}{s+1} = \frac{e^{ts+\cancel{t}-\cancel{t}}}{s+1} \Rightarrow \frac{e^{st}}{s+1}$$

$$= \lambda e^{st} \qquad \text{if } \operatorname{Re} s > -1$$

Thus, e^{st} is the eigenfunction of the system and the associated eigenvalue is

$$\lambda = \frac{1}{s+1}$$

c) The eigenvalue associated with e^{st} is given by

$$\lambda = H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{-s\tau} d\tau$$

$$= \int_{0}^{\infty} e^{-(s+1)\tau} d\tau = \frac{-1 e^{-(s+1)\tau}}{s+1} \Big|_{0}^{\infty}$$

$$= \left[\frac{-e^{-(s+1)\infty}}{s+1} + \frac{e^{-(s+1)0}}{s+1} \right] \Rightarrow \frac{1}{s+1} \text{ if } \operatorname{Re} s > -1.$$

Which is same as λ in the part b.