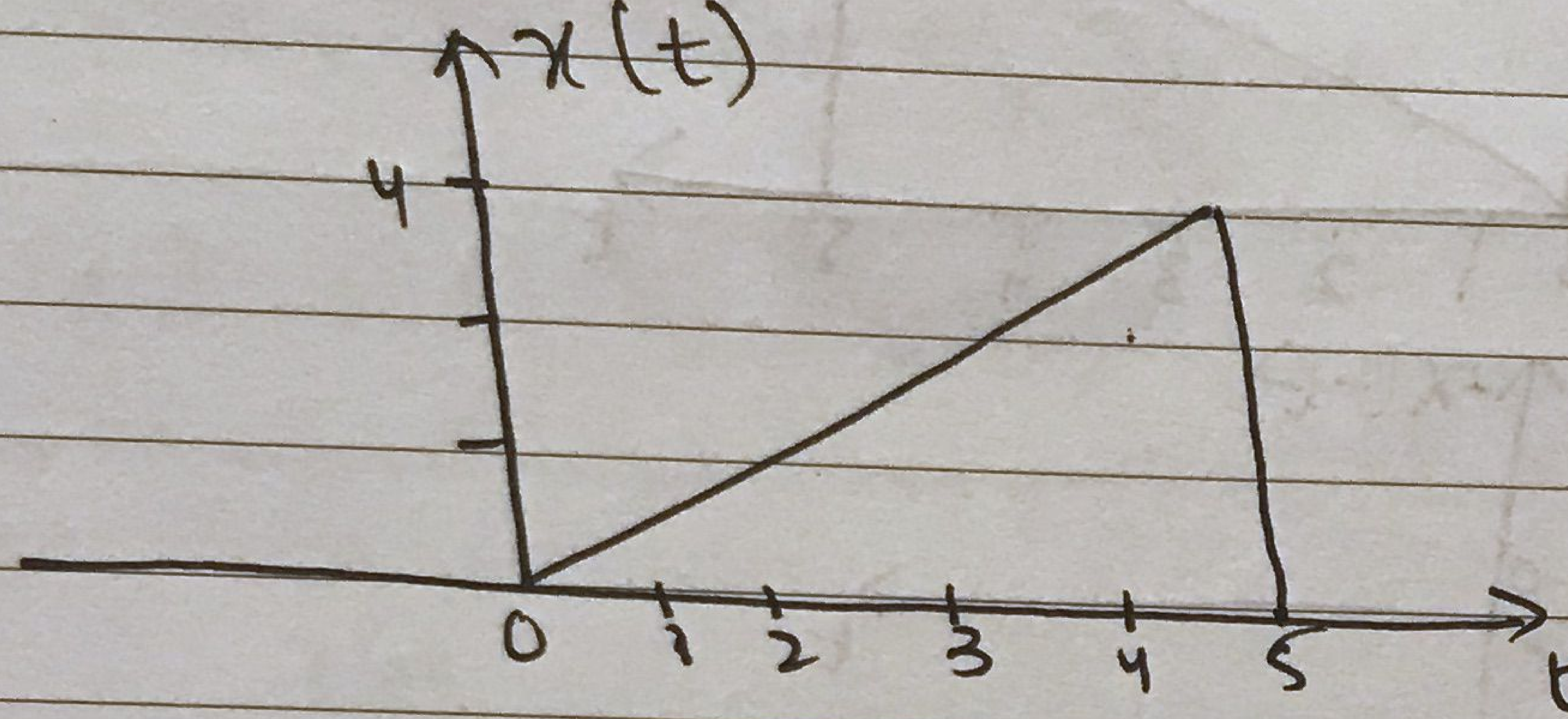
Quiz # 2

Solution

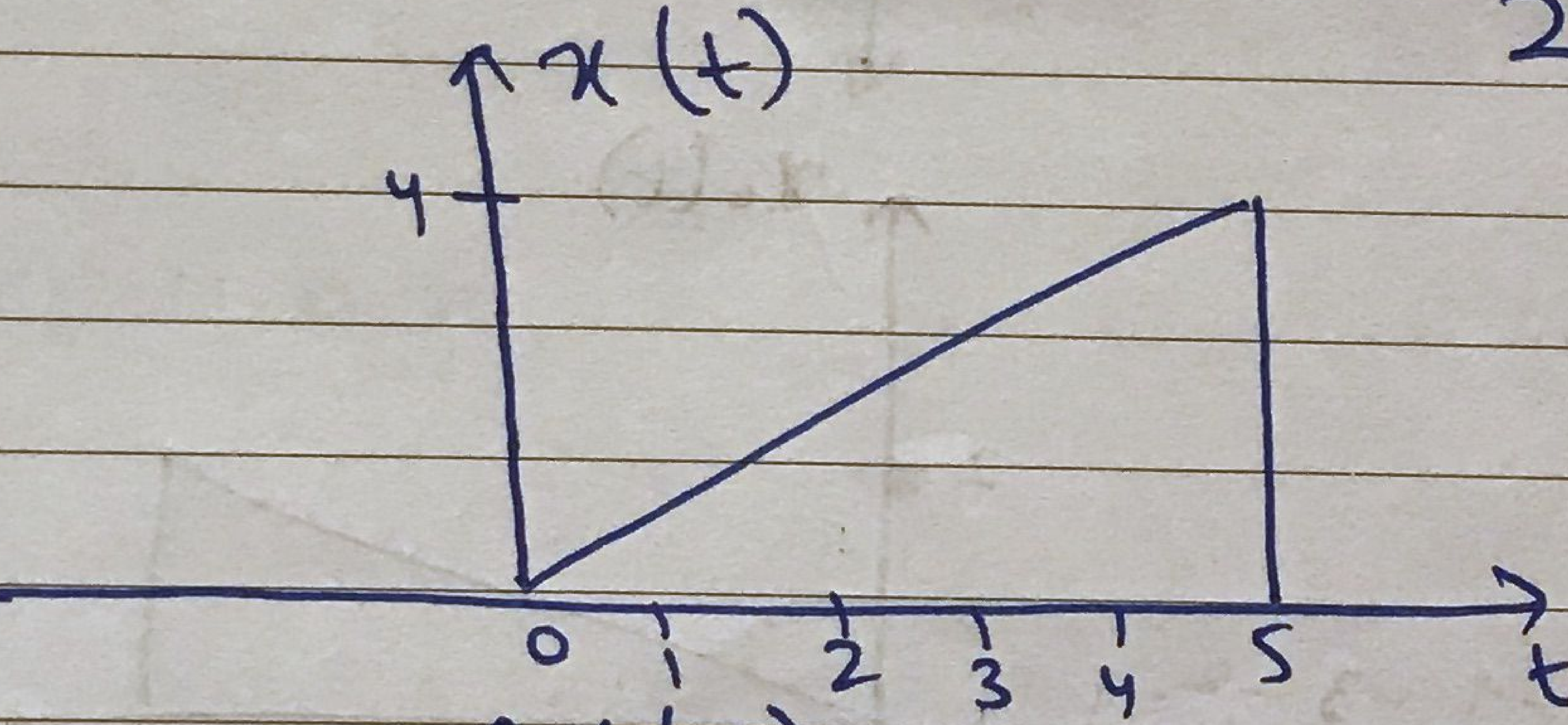
Q # 1:

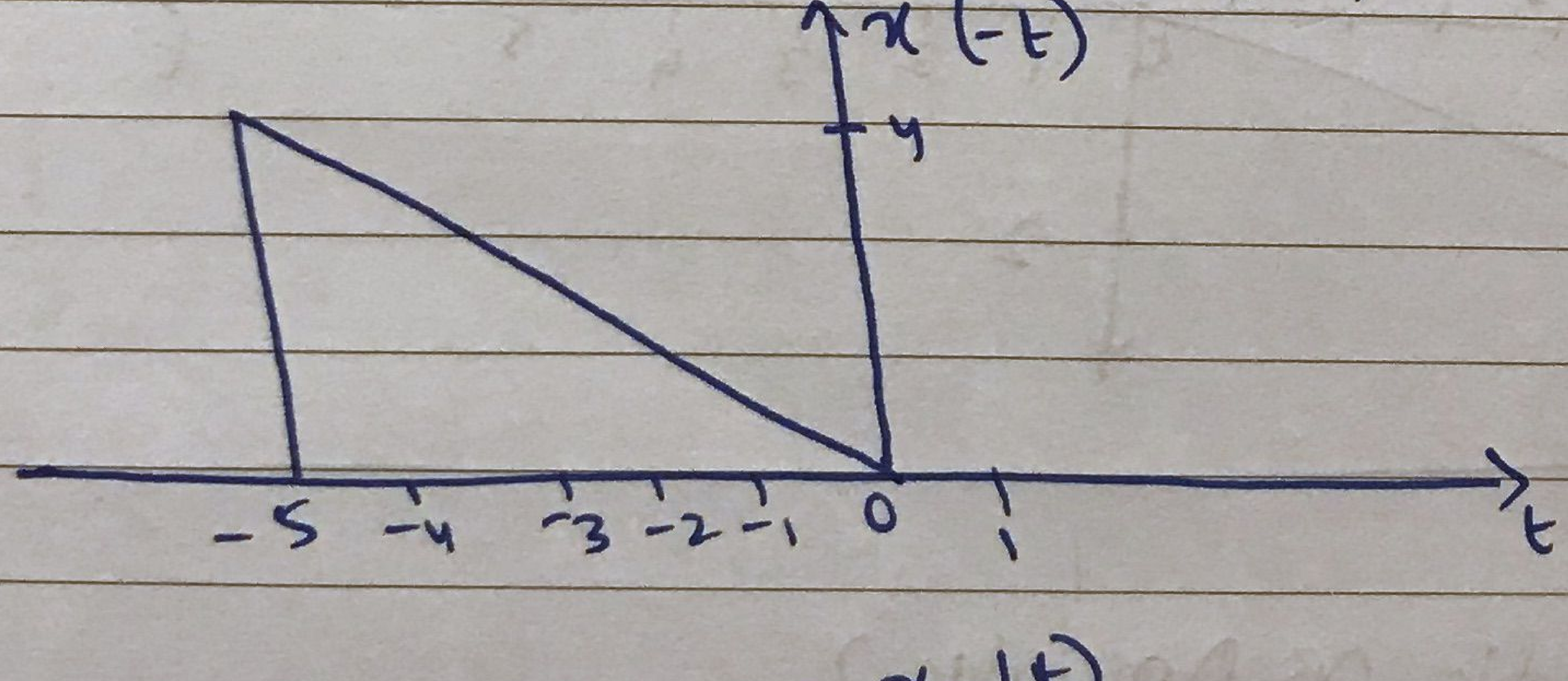
Even and odd component = ?

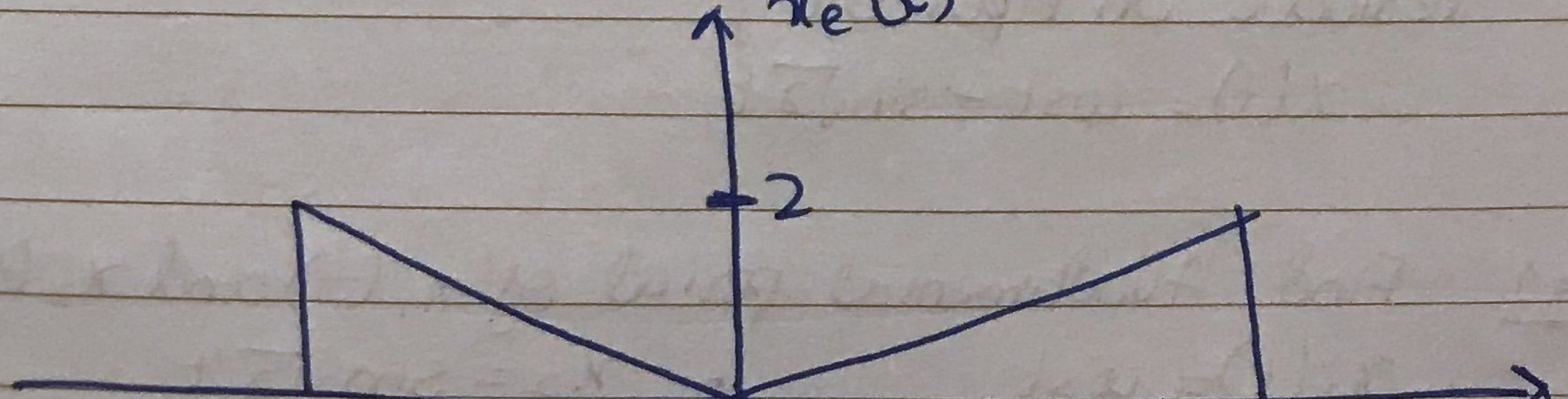


Sol:

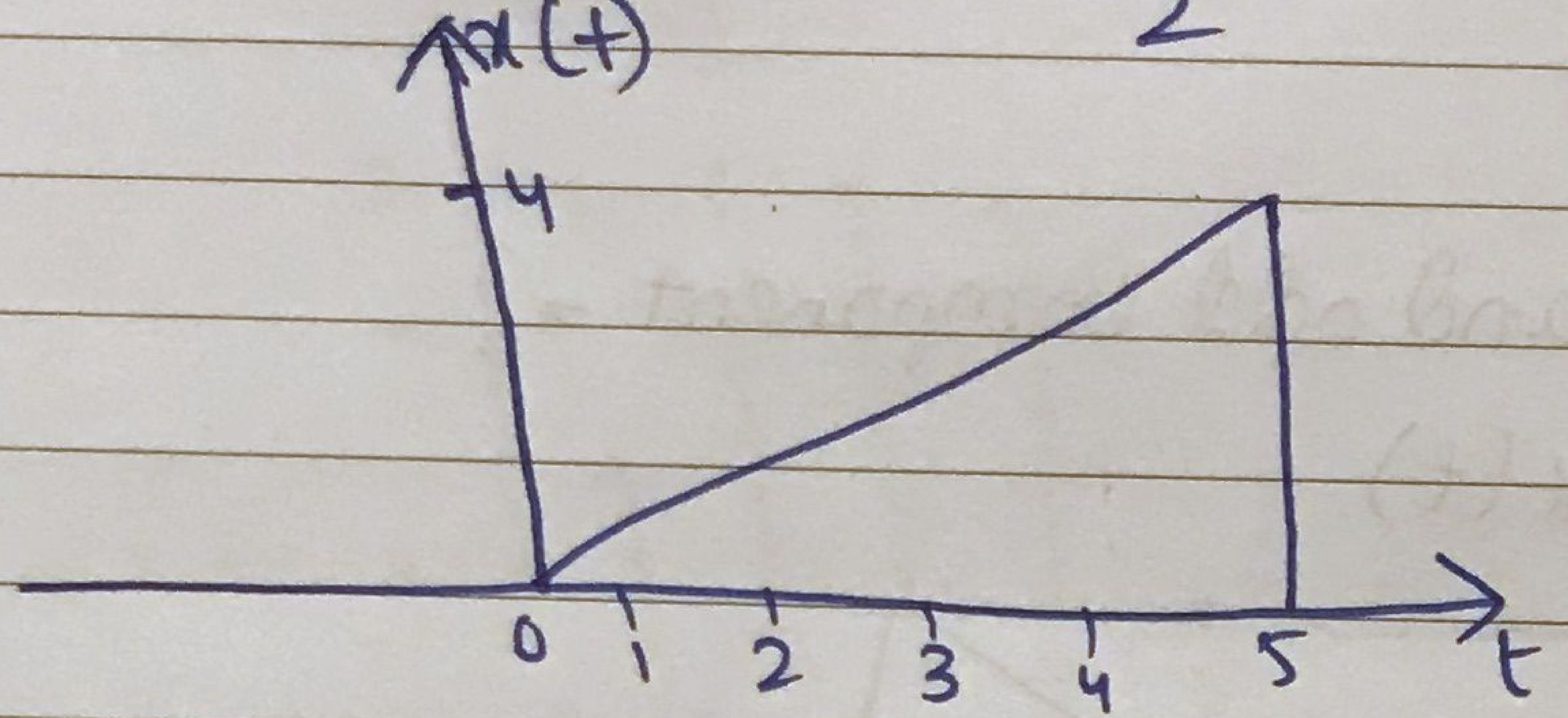
Even component $\Rightarrow x_e[t] = \dfrac{x[t] + x[-t]}{2}$

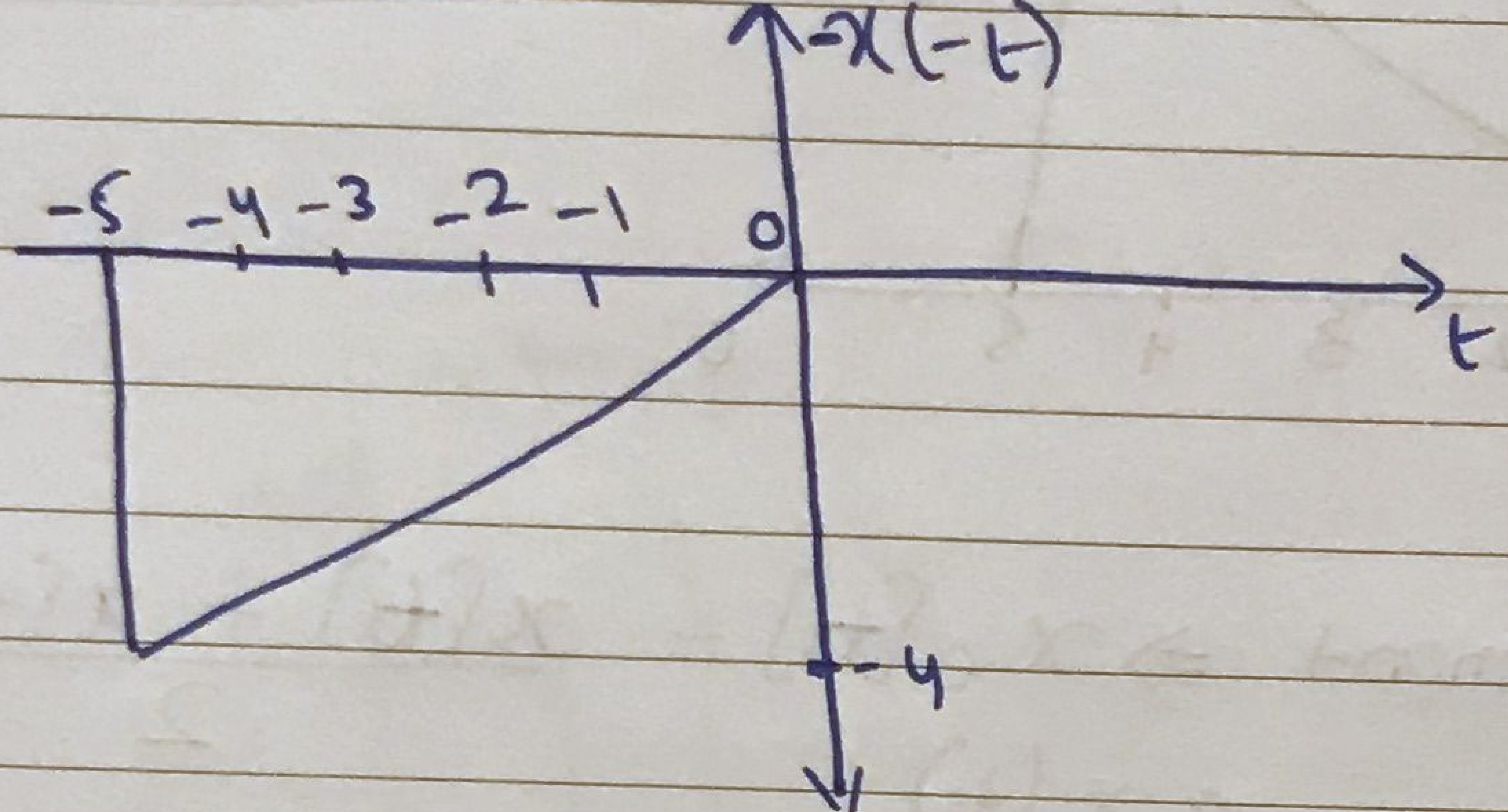


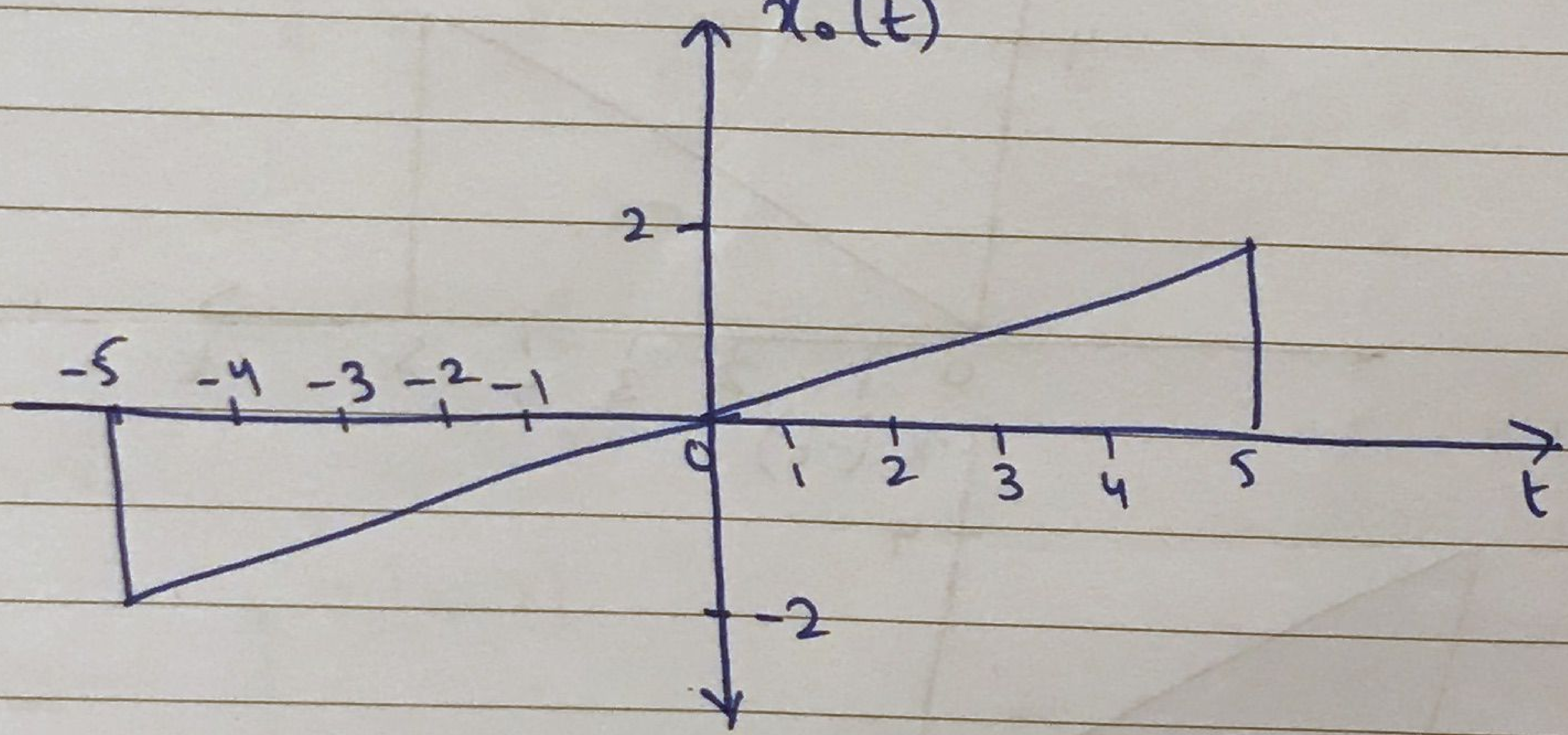




Odd Component: $x_o(t) = \dfrac{x(t) - x(-t)}{2}$







Q#2:

Periodic OR Aperiodic?

$x(t) = \cos t + \sin\sqrt{2}\, t$

Sol:

Step #1 Find fundamental period of $x_1(t)$ and $x_2(t)$

$x_1(t) = \cos t$, $x_2 = \sin\sqrt{2}\, t$

$T_1 = \dfrac{2\pi}{\omega_1}$ $\because \omega_1 = 1$, $T_2 = \dfrac{2\pi}{\omega_2}$ $\because \omega_2 = \sqrt{2}$

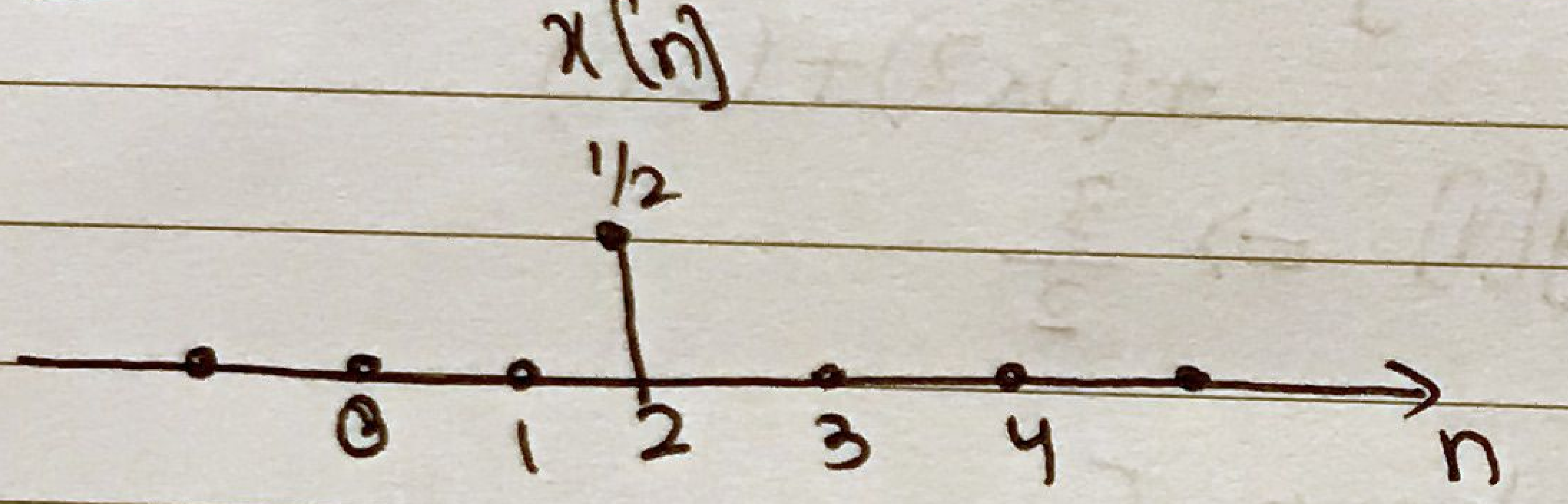
$T_1 = \dfrac{2\pi}{1} \Rightarrow 2\pi$, $T_2 \Rightarrow \dfrac{2\pi}{\sqrt{2}}$

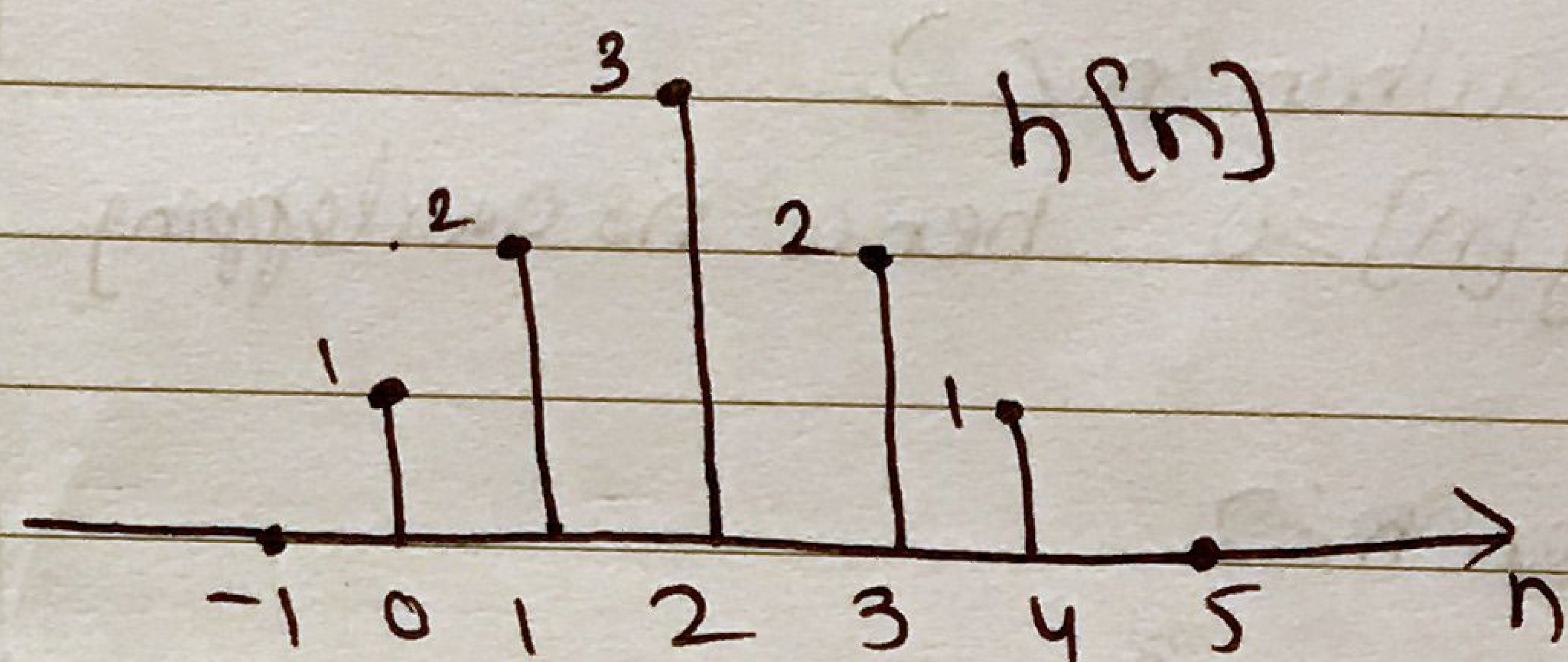
Step #2: Find rationality of $\frac{T_1}{T_2}$

$$\frac{T_1}{T_2} = \frac{2\pi}{2\pi/\sqrt{2}} = \frac{\cancel{2\pi}}{\cancel{2\pi}} \times \sqrt{2} \Rightarrow \sqrt{2} \text{ irrational.}$$

Hence, the ratio of $\frac{T_1}{T_2}$ is irrational, then the given signal $x(t)$ is Aperiodic.

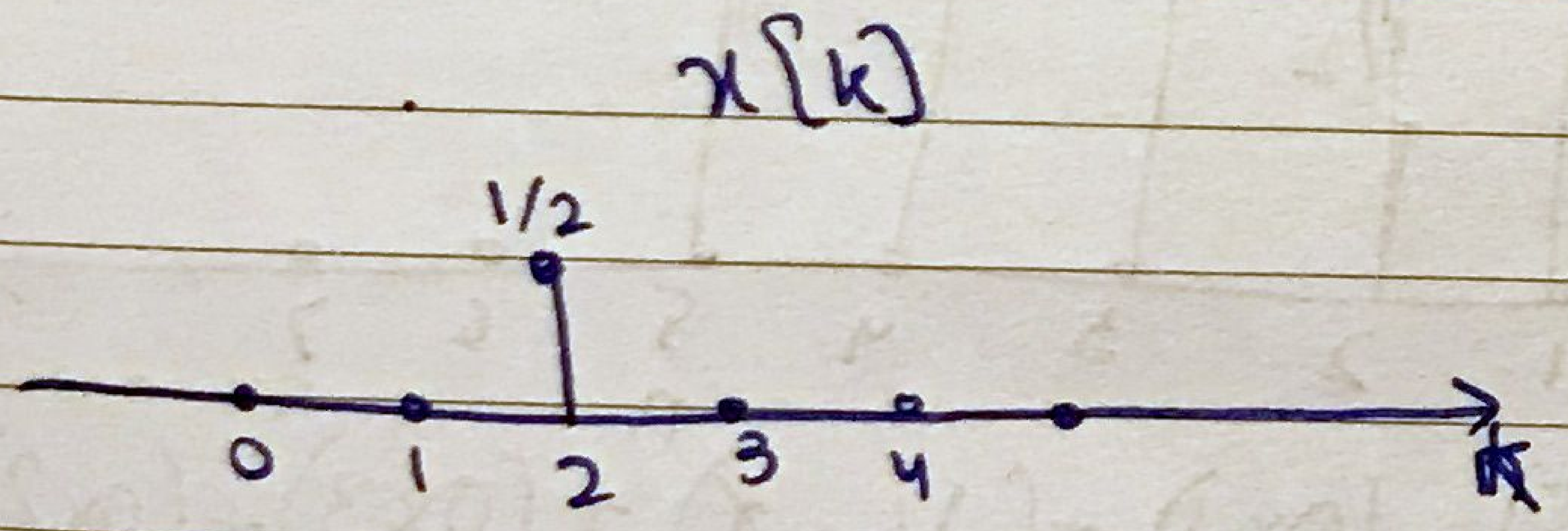
Q#3:

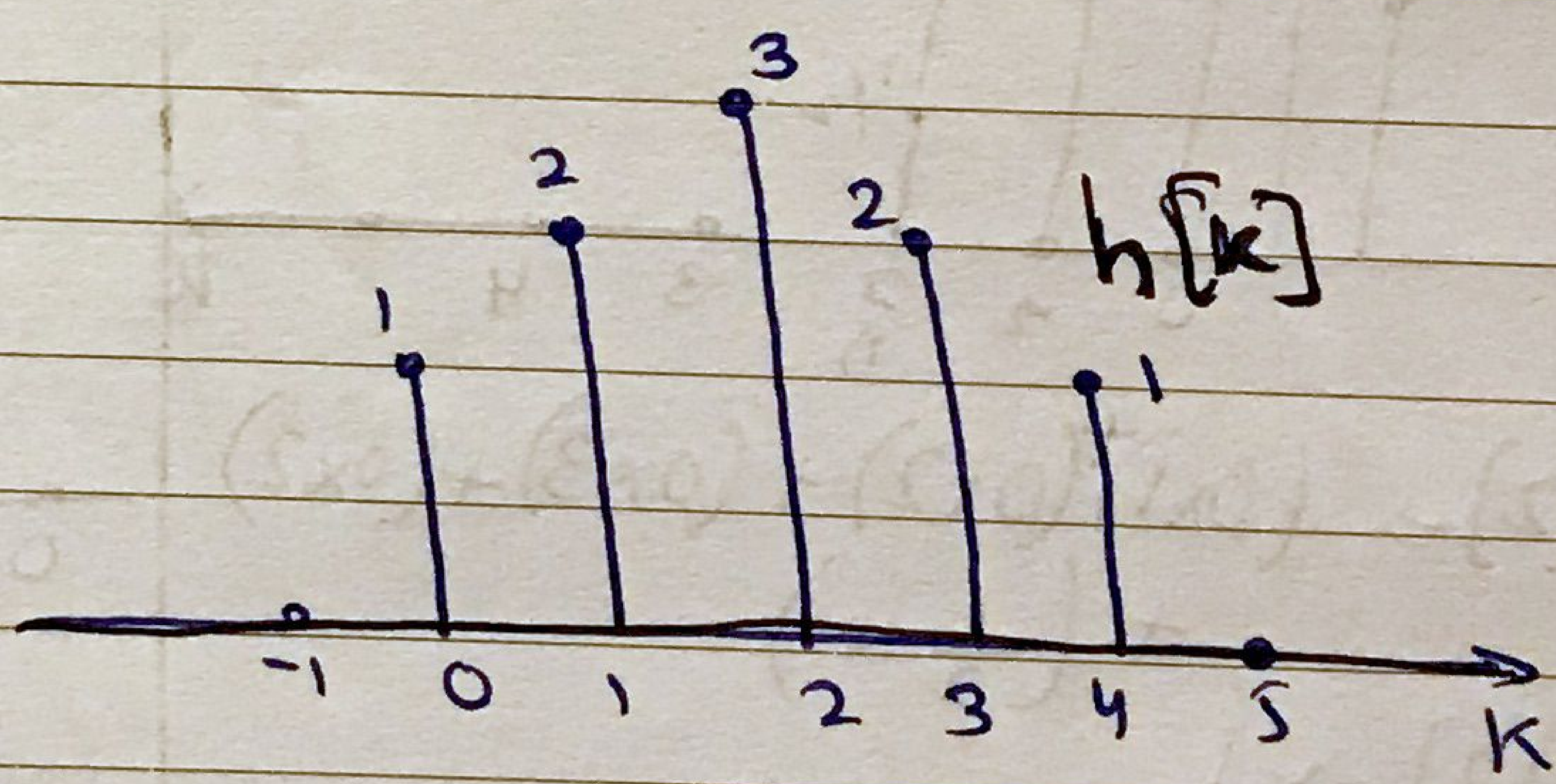




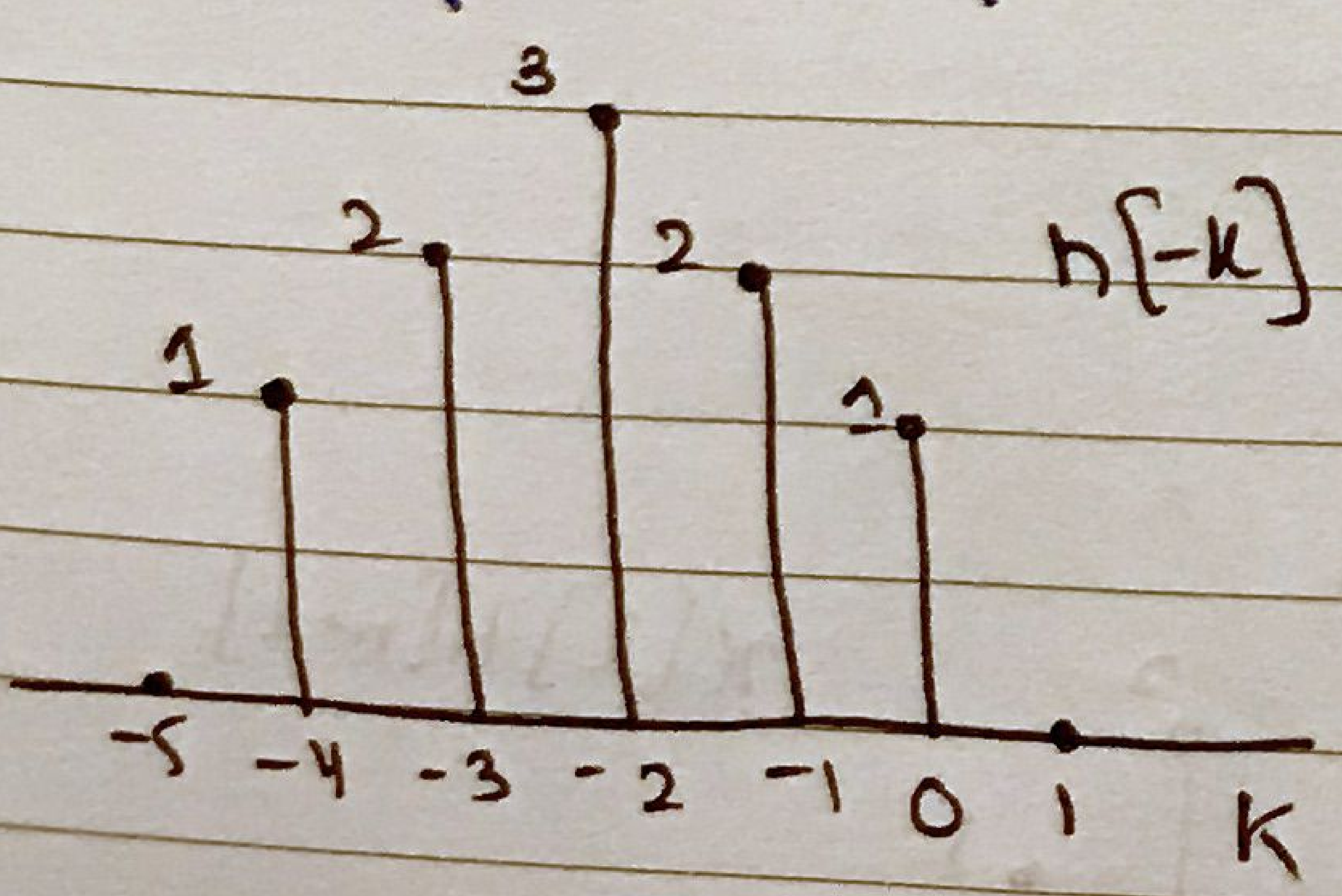
Sol:

Step #1: Replace $n \to k$

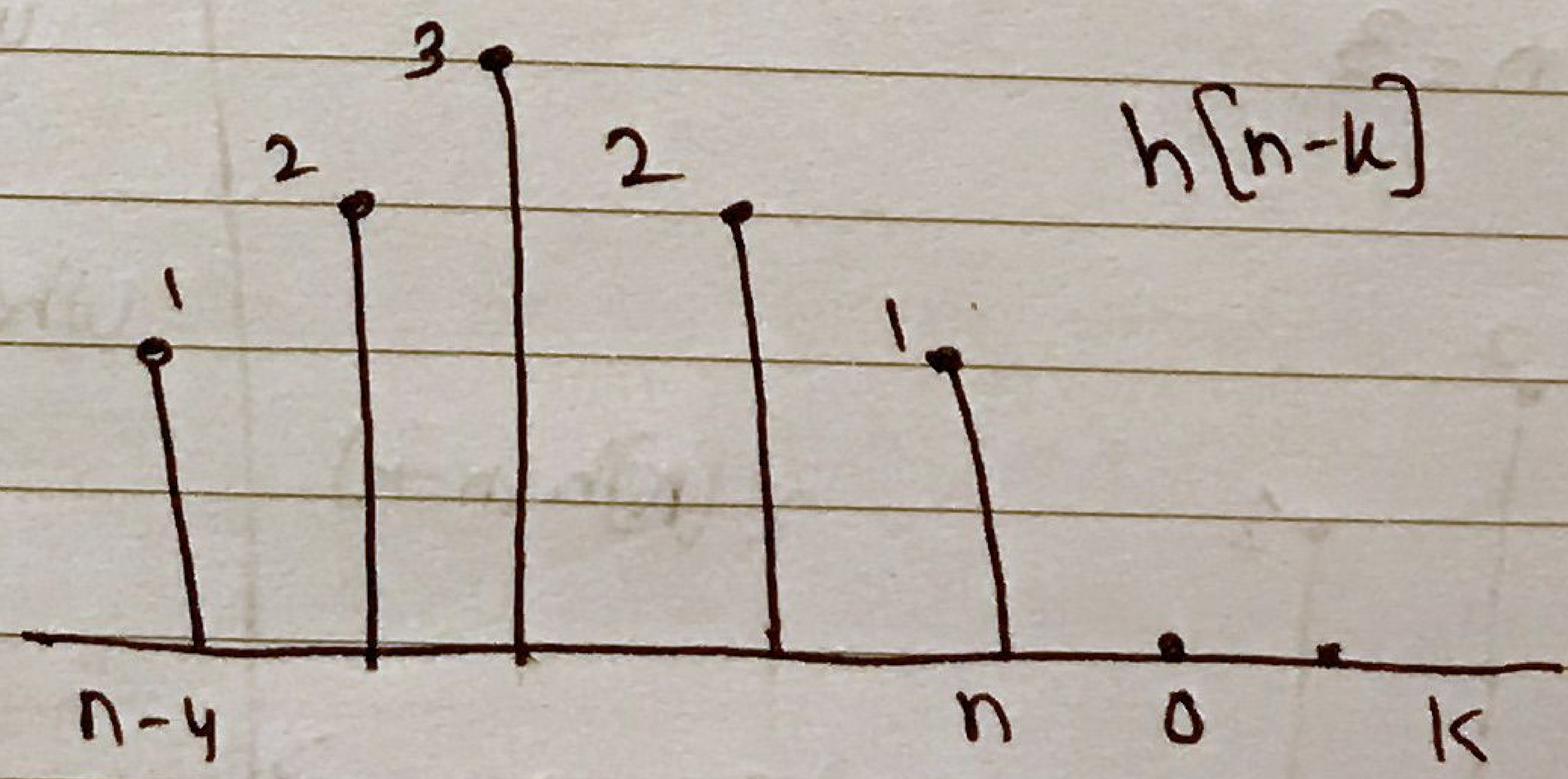




Step #2: Flip and shift $h[k]$

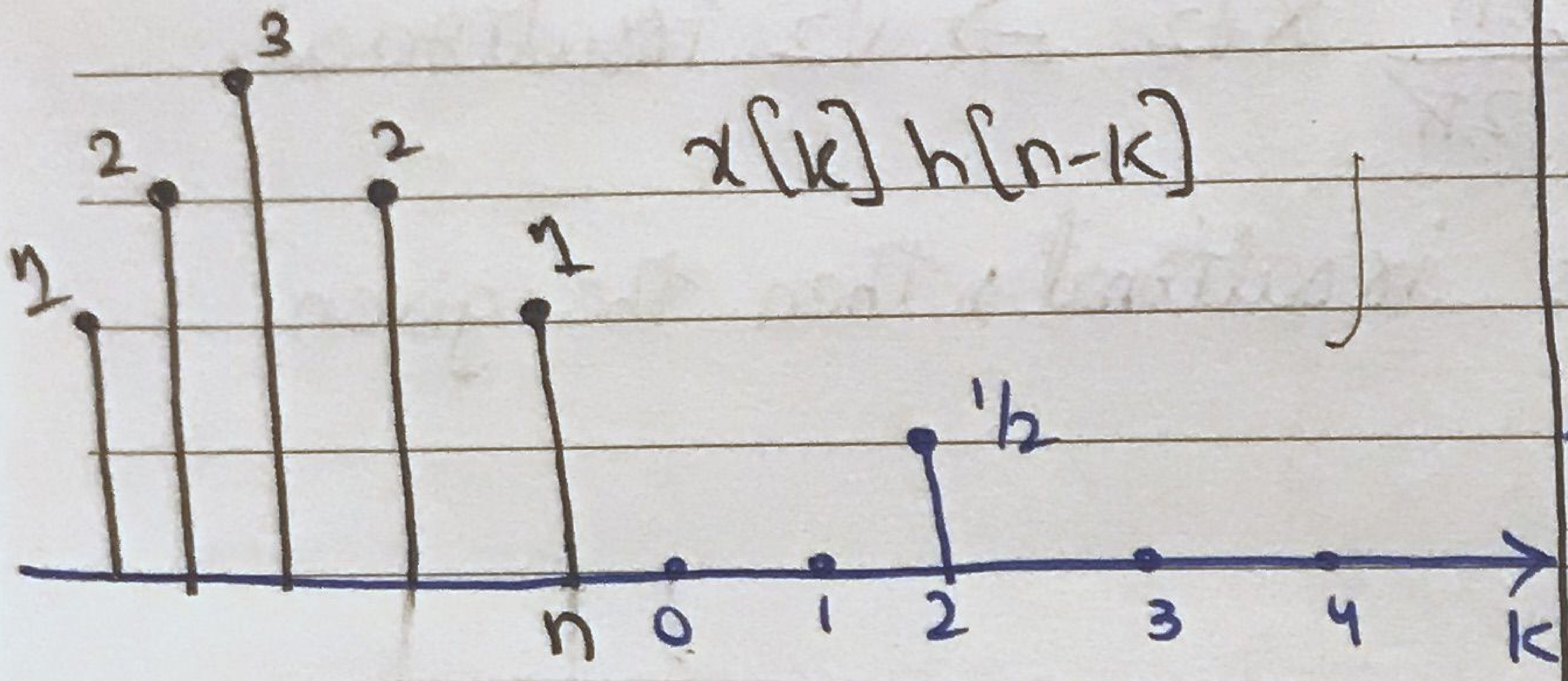


$\Rightarrow$



Step#3:

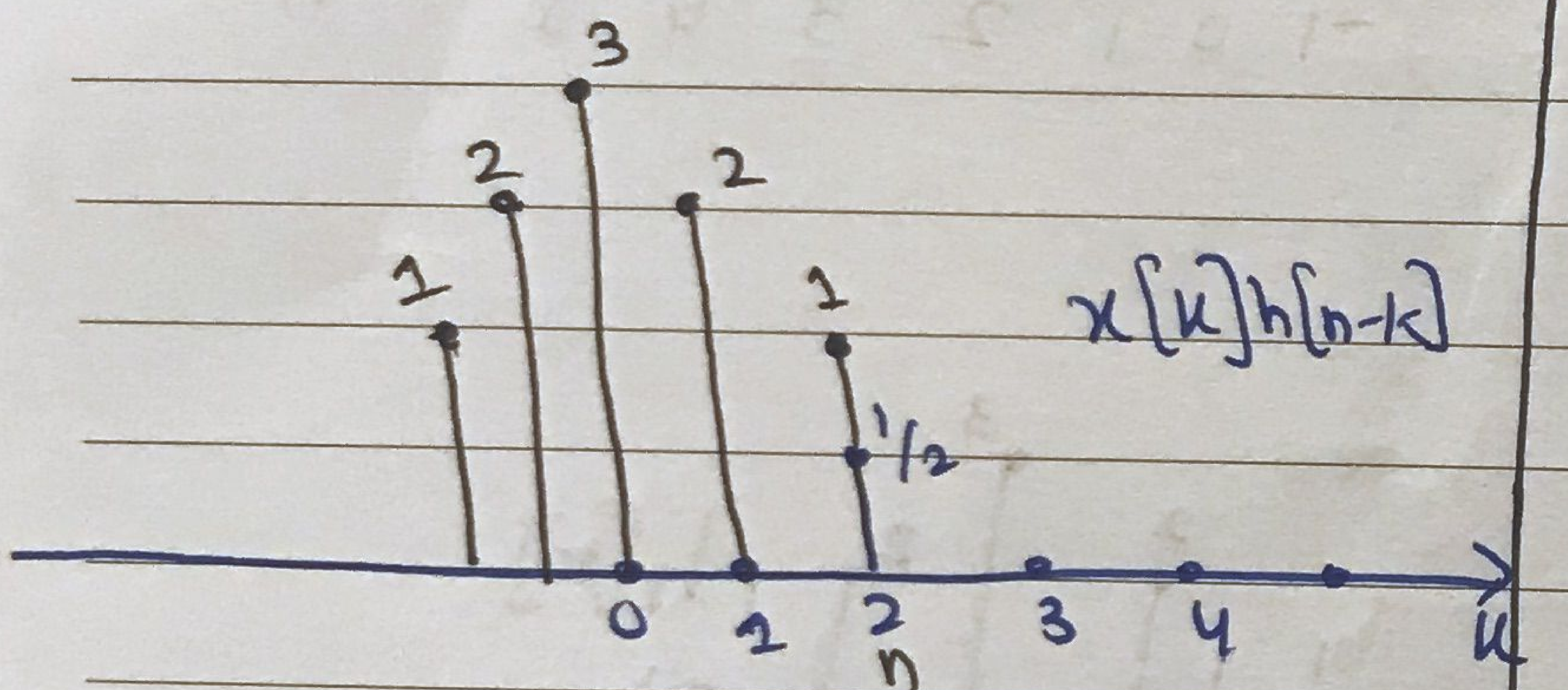
Now Convolve $x[k]$ and $h[n-k]$



when $n < 2$

$y[n] = 0$ hence no overlapping.

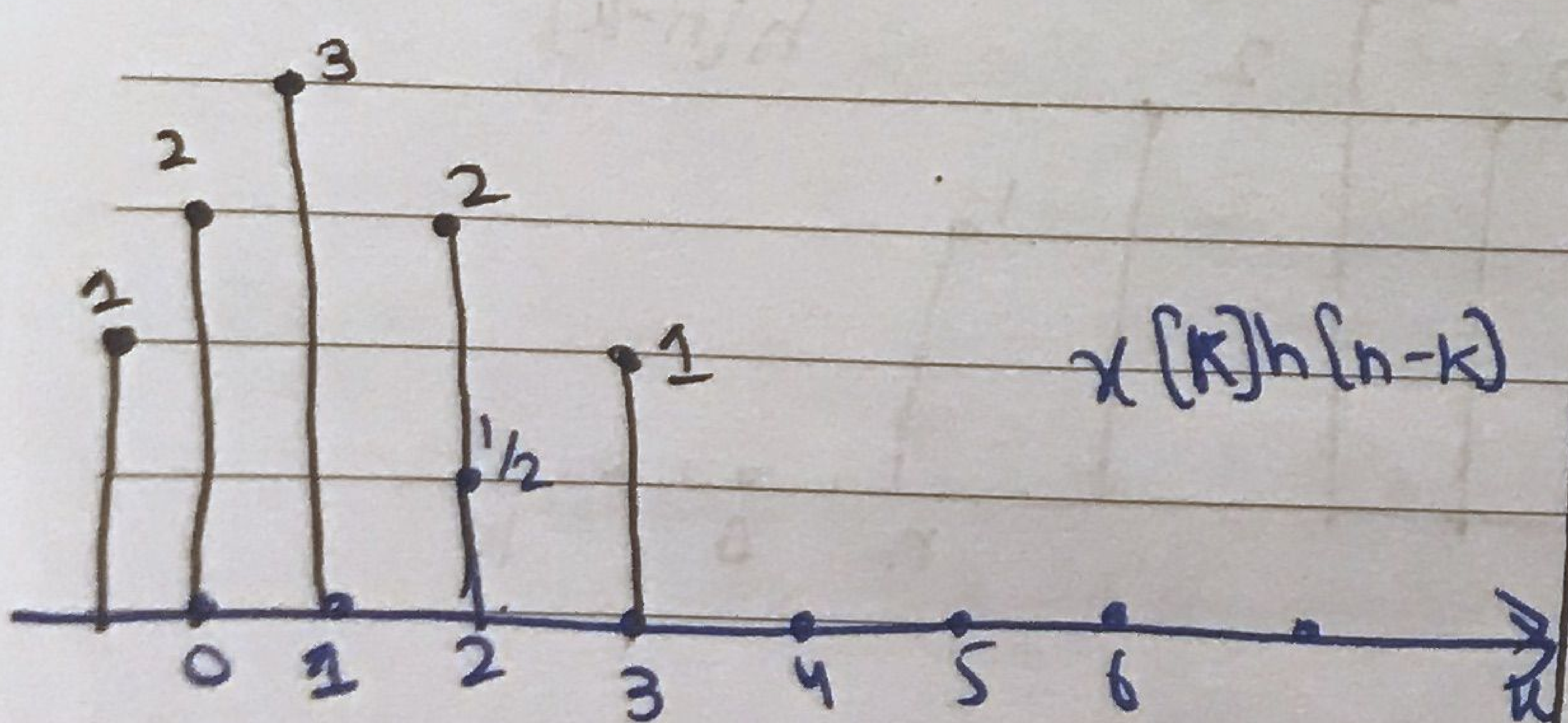
when $n = 2$



$y[2] = (0\times1)+(0\times2)+(0\times3)+(0\times2) + (\frac{1}{2}\times1)$

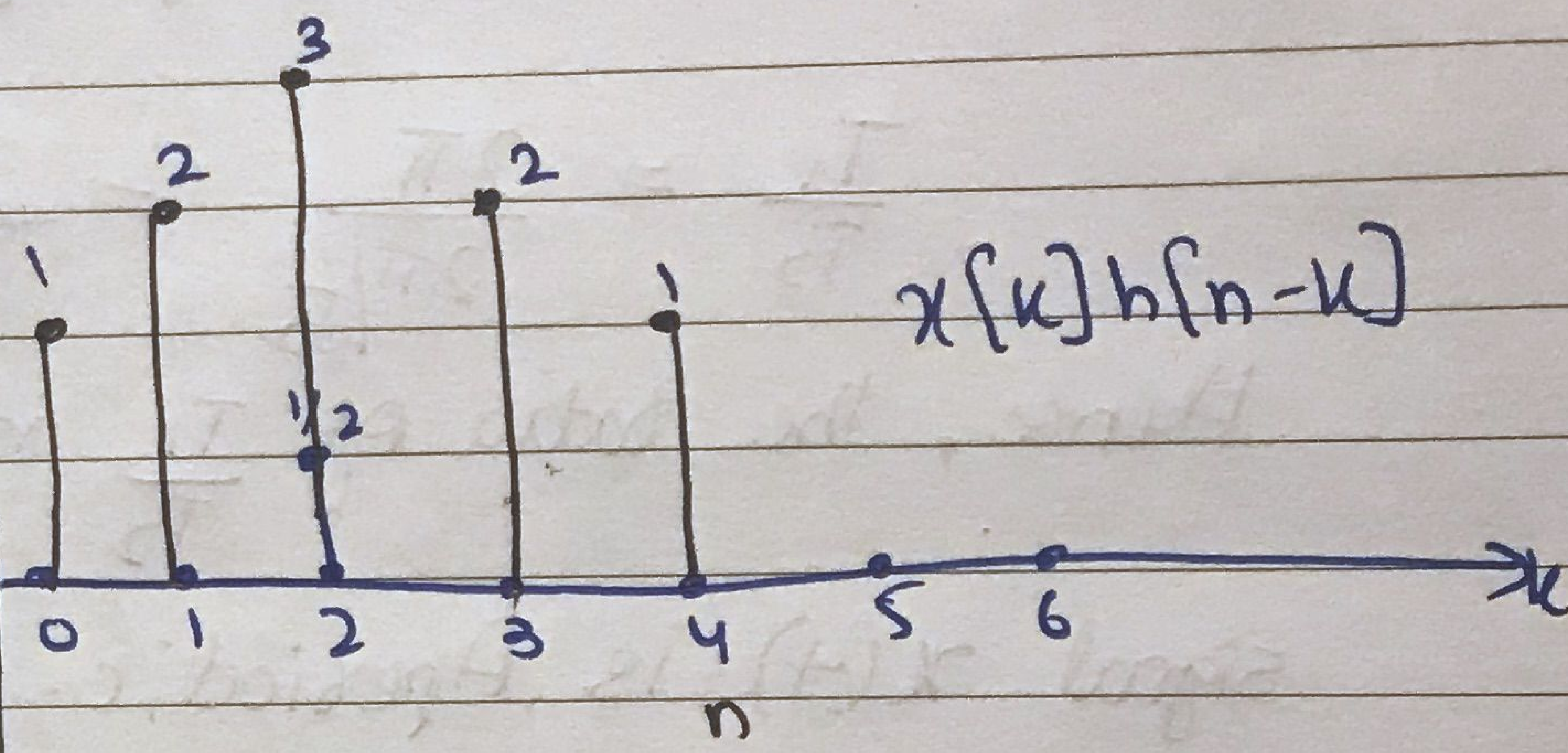
$y[2] \Rightarrow \frac{1}{2}$

when $n = 3$



$y[3] = (0\times1)+(0\times2)+(0\times3) + (2\times\frac{1}{2}) + (0\times1) \Rightarrow 1$

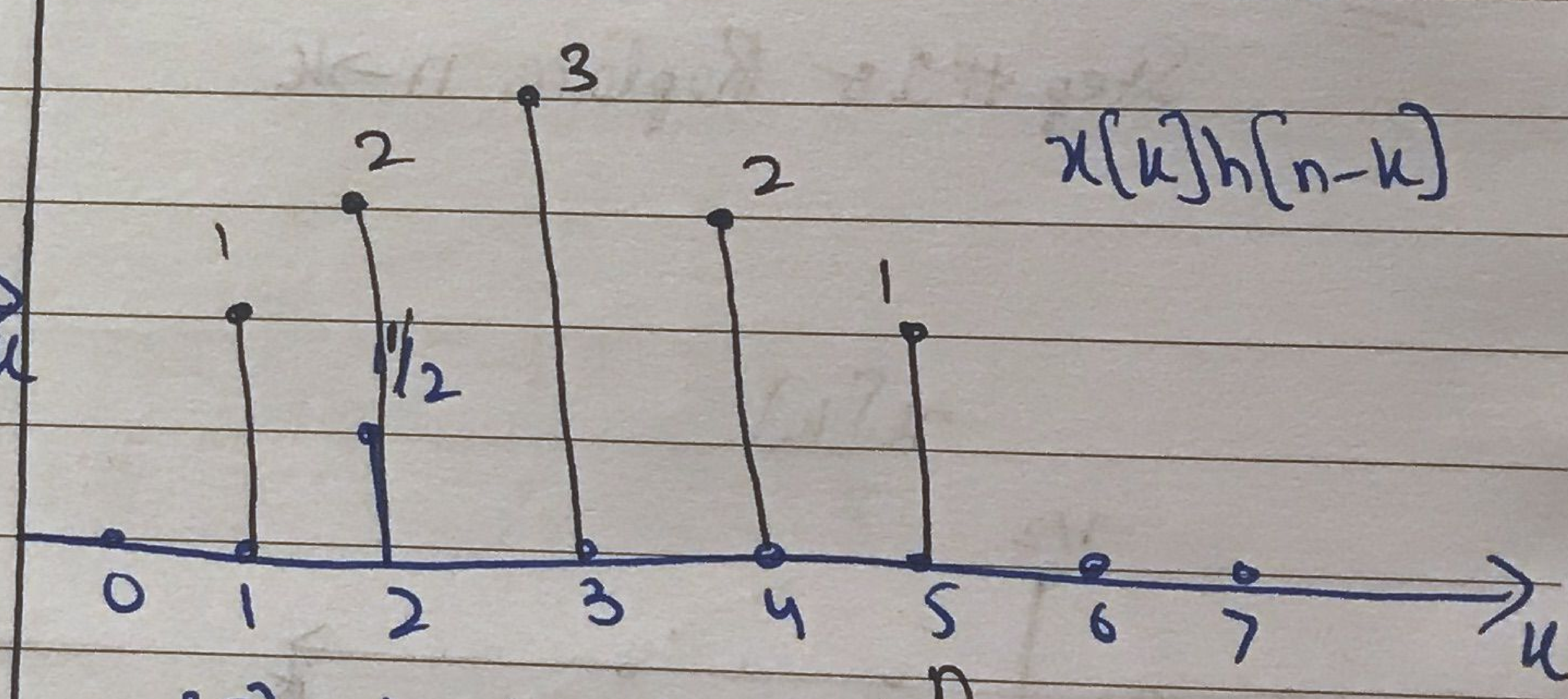
when $n = 4$



$y[4] = (0\times1) + (0\times1) + (\frac{1}{2}\times3) + (0\times2) + (0\times1)$

$y[4] \Rightarrow \frac{3}{2}$

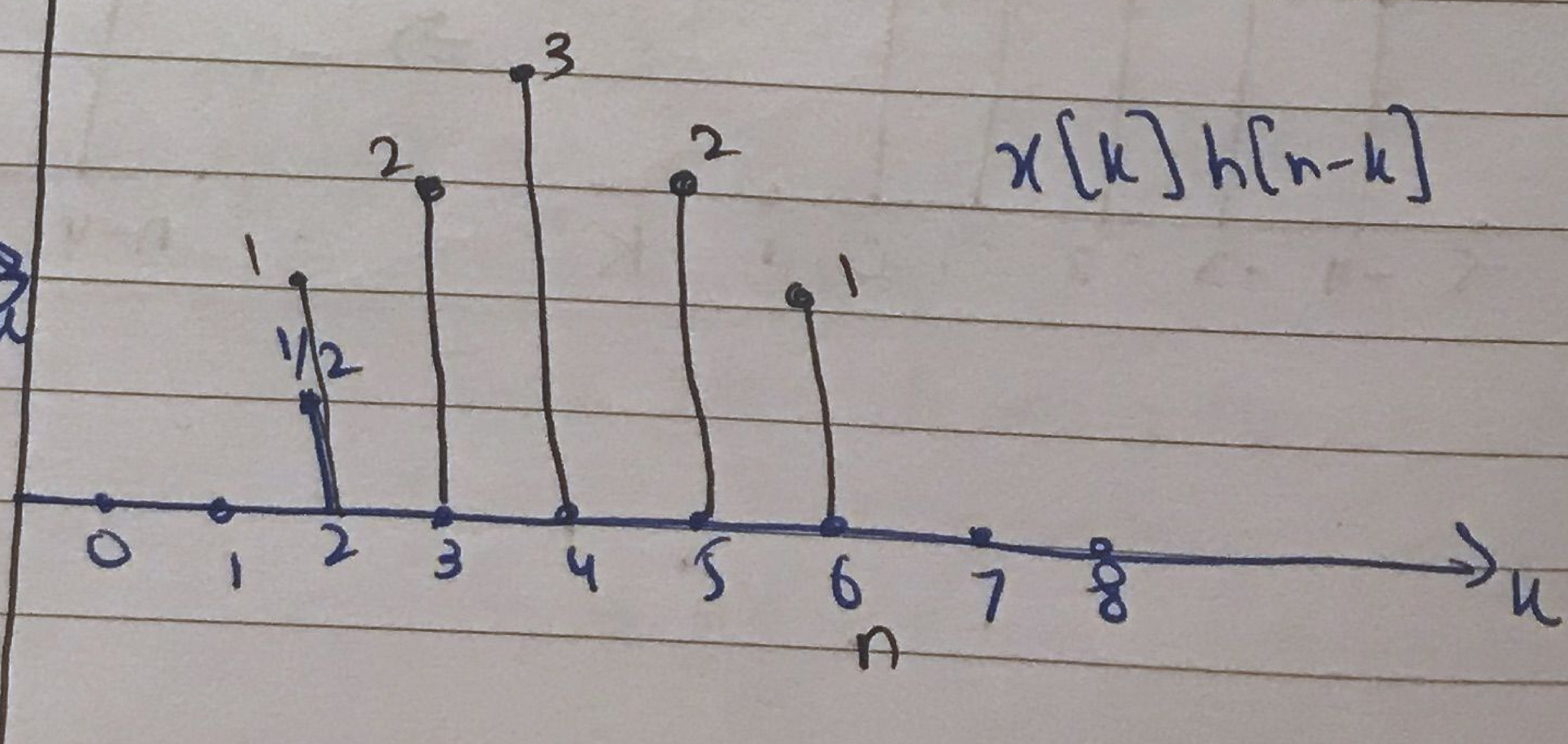
when $n = 5$



$y[5] = (0\times1) + (\frac{1}{2}\times2) + (0\times3) + (0\times2) + 0$

$y[5] \Rightarrow 1$

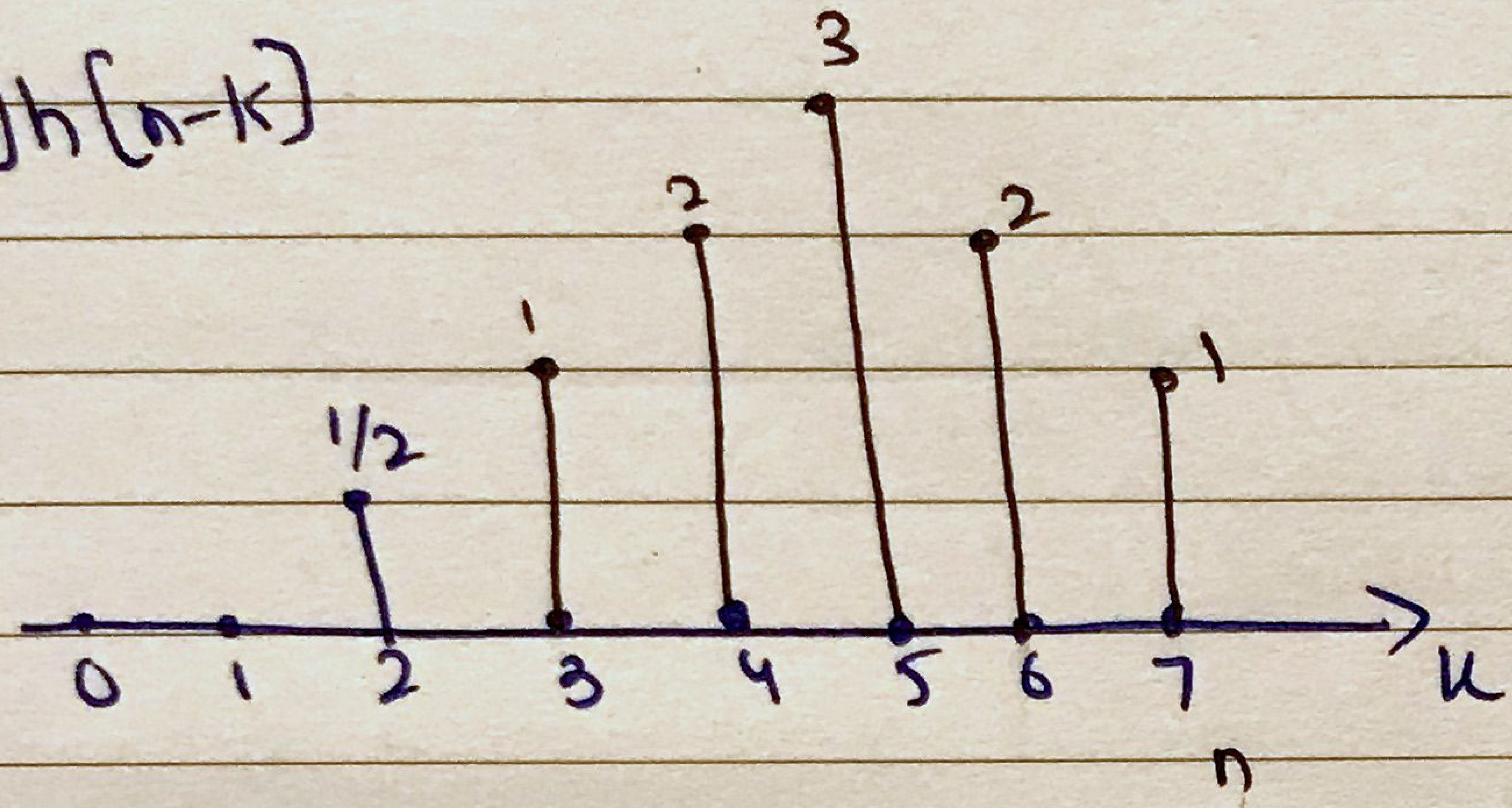
when $n = 6$



$$y[6] = \left(\tfrac{1}{2} \times 1\right) + (0 \times 1) + (0 \times 1) + (0 \times 1) + (0 \times 1) \Rightarrow \frac{1}{2}$$

when $n > 6$

$x[k]h[n-k]$



$y[n] = 0$, Hence no overlapping

$$y[n] = \begin{cases} 0 & n < 2 \\ 1/2 & n = 2 \\ 1 & n = 3 \\ 3/2 & n = 4 \\ 1 & n = 5 \\ 1/2 & n = 6 \\ 0 & n > 6 \end{cases}$$