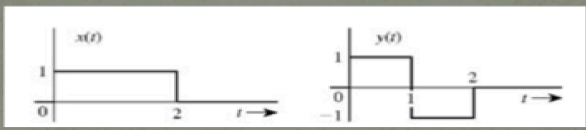
Example #3

- Find the energies of the pair of signals x(t) and y(t) shown below. Sketch and find the energies of signals of x(t)+y(t) and x(t)-y(t).



(a) $E_x = \int_0^2 (1)^2 dt = 2.$ $\quad E_y = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt = 2$

$$E_{x+y} = \int_0^1 (2)^2 dt = 4, \; E_{x-y} = \int_1^2 (2)^2 dt = 4$$

Therefore $E_{x \pm y} = E_x + E_y$.

Example #4

- Determine whether the following signal is power signal or not:

$$x[n] = 2e^{j3\pi}$$

(*f*) Since $|x[n]| = |2e^{j3n}| = 2|e^{j3n}| = 2,$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 2^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} 4(2N+1) = 4 < \infty$$

Thus, $x[n]$ is a power signal.

Example #5

- Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period:
  - (1): $x(t) = \cos t + \sin\sqrt{2}t$
  - (2): $x[n] = e^{j(\pi/4)n}$

(*d*) $x(t) = \cos t + \sin\sqrt{2}\,t = x_1(t) + x_2(t)$

where $x_1(t) = \cos t = \cos \omega_1 t$ is periodic with $T_1 = 2\pi/\omega_1 = 2\pi$ and $x_2(t) = \sin\sqrt{2}\,t = \sin \omega_2 t$ is periodic with $T_2 = 2\pi/\omega_2 = \sqrt{2}\,\pi$. Since $T_1/T_2 = \sqrt{2}$ is an irrational number, $x(t)$ is nonperiodic.

(g) $x[n] = e^{j(\pi/4)n} = e^{j\Omega_0 n} \rightarrow \Omega_0 = \frac{\pi}{4}$

Since $\Omega_0/2\pi = \frac{1}{8}$ is a rational number, $x[n]$ is periodic, and by Eq. (1.55) the fundamental period is $N_0 = 8$.

Example #6

- Find the even and odd components of $x(t) = e^{jt}$.

Let $x_e(t)$ and $x_o(t)$ be the even and odd components of e^{jt}, respectively.

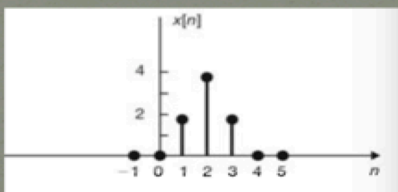
$$e^{jt} = x_e(t) + x_o(t)$$

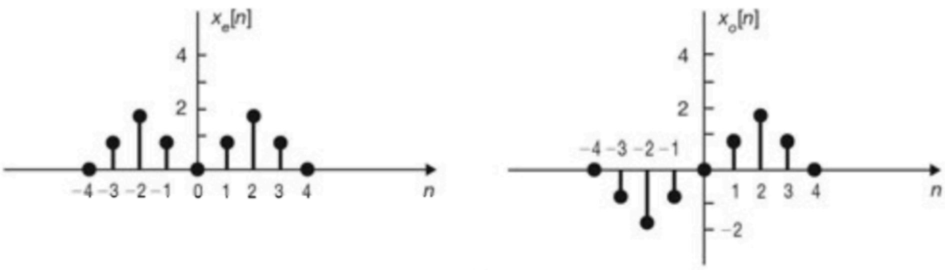
From Eqs. (1.5) and (1.6) and using Euler's formula, we obtain

$$x_e(t) = \frac{1}{2}(e^{jt} + e^{-jt}) = \cos t$$
$$x_o(t) = \frac{1}{2}(e^{jt} - e^{-jt}) = j\sin t$$

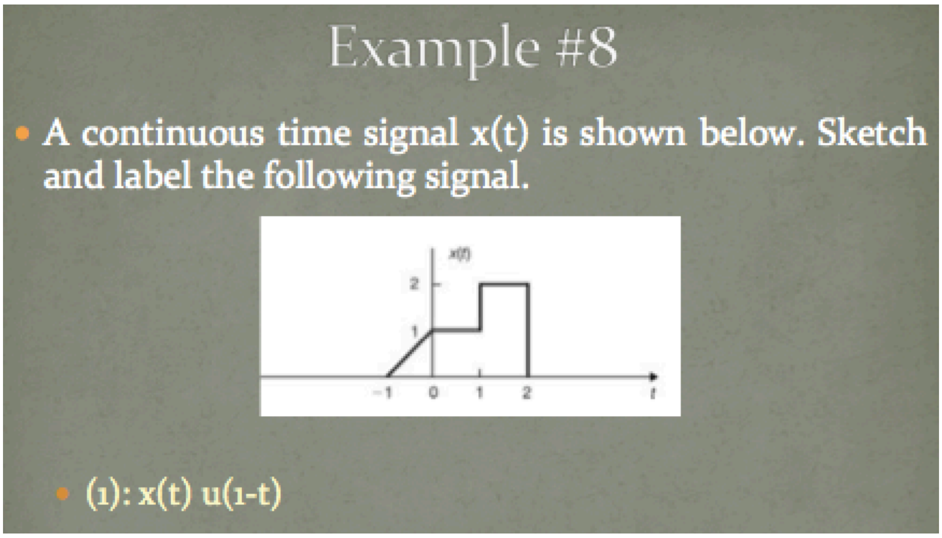
Example #7

- Sketch and label the even and odd components of the signal shown below:





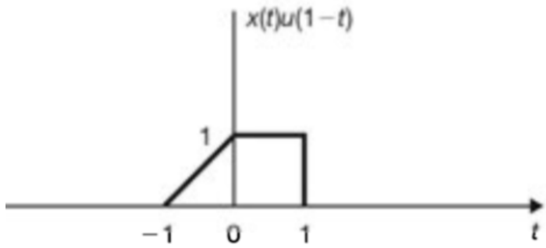
(d)



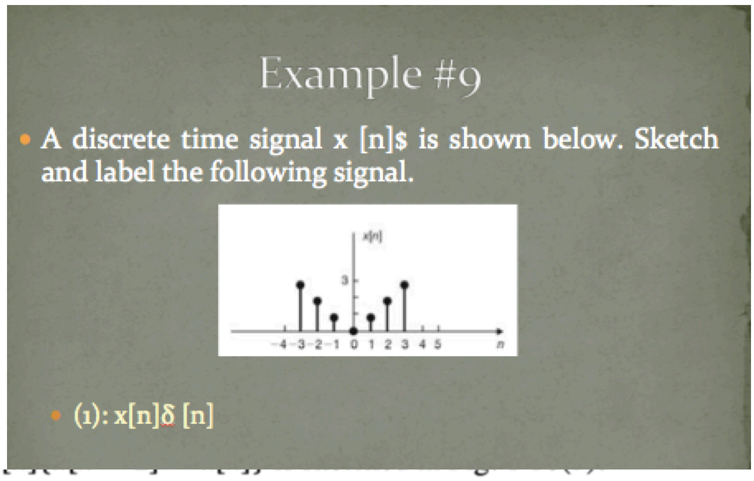
(*a*) By definition (1.19)

$$u(1-t)=\begin{cases}1 & t<1\\ 0 & t>1\end{cases}$$

and $x(t)u(1-t)$ is sketched in Fig. 1-28(*a*).



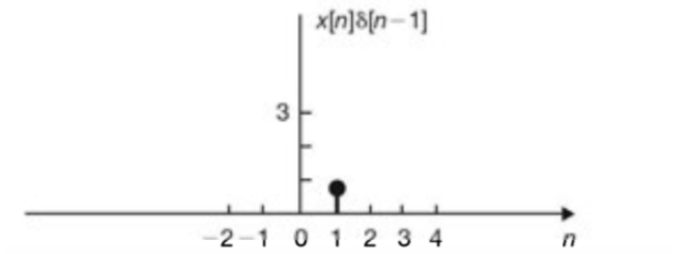
(*a*)



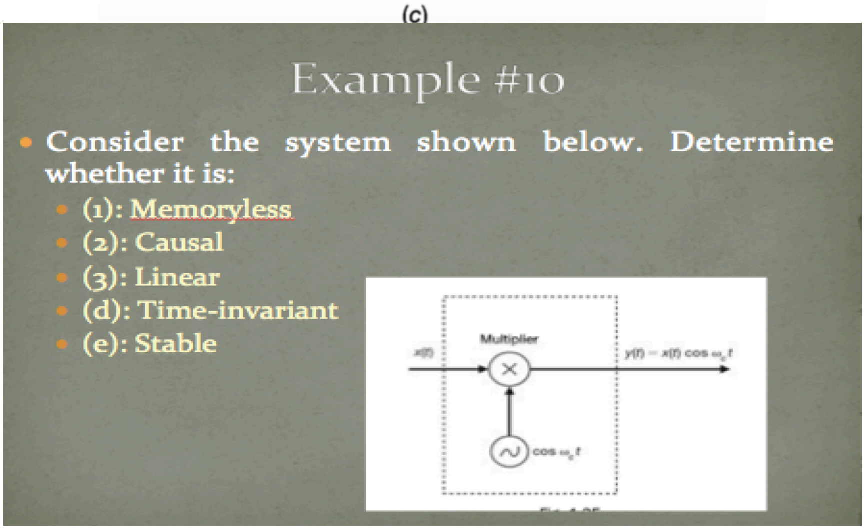
(*c*) By definition (1.48)

$$x[n]\delta[n-1]=x[1]\delta[n-1]=\delta[n-1]=\begin{cases}1 & n=1\\ 0 & n\neq 1\end{cases}$$

which is sketched in Fig. 1-30(*c*).



(c)



(*a*) From Fig. 1-35 we have

$$y(t) = \mathbf{T}\{x(t)\} = x(t)\cos\omega_c t$$

Since the value of the output $y(t)$ depends on only the present values of the input $x(t)$, the system is memoryless.

(*b*) Since the output $y(t)$ does not depend on the future values of the input $x(t)$, the system is causal.

(*c*) Let $x(t) = \alpha_1 x(t) + \alpha_2 x(t)$. Then

$$\begin{aligned} y(t) &= \mathbf{T}\{x(t)\} = [\alpha_1 x_1(t) + \alpha_2 x_2(t)]\cos\omega_c t \\ &= \alpha_1 x_1(t)\cos\omega_c t + \alpha_2 x_2(t)\cos\omega_c t \\ &= \alpha_1 y_1(t) + \alpha_2 y_2(t) \end{aligned}$$

Thus, the superposition property (1.68) is satisfied and the system is linear.

(*d*) Let $y_1(t)$ be the output produced by the shifted input $x_1(t) = x(t - t_0)$. Then

$$y_1(t) = \mathbf{T}\{x(t - t_0)\} = x(t - t_0)\cos\omega_c t$$

But

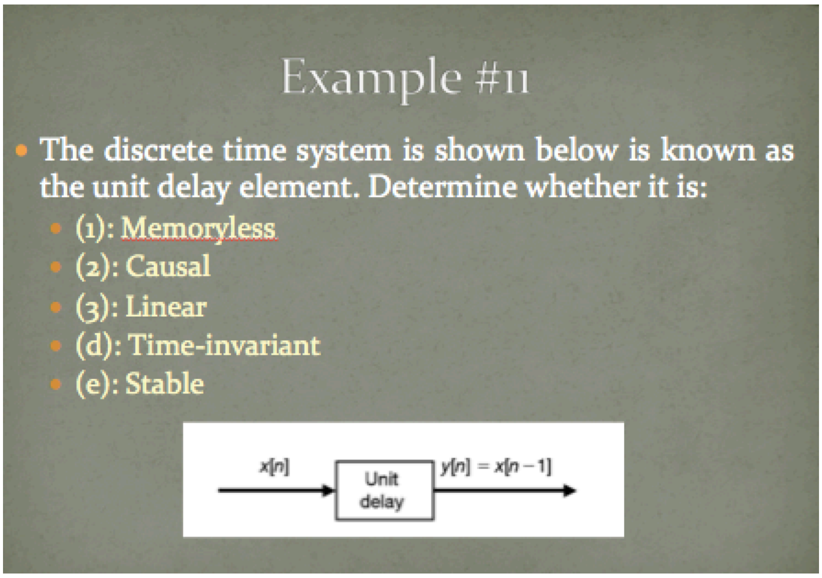
$$y(t - t_0) = x(t - t_0)\cos\omega_c(t - t_0) \neq y_1(t)$$

Hence, the system is not time-invariant.

(*e*) Since $|\cos\omega_c t| \leq 1$, we have

$$|y(t)| = |x(t)\cos\omega_c t| \leq |x(t)|$$

Thus, if the input $x(t)$ is bounded, then the output $y(t)$ is also bounded and the system is BIBO stable.



(*a*) The system input-output relation is given by

$$y[n] = \mathbf{T}\{x[n]\} = x[n-1]$$

Since the output value at n depends on the input values at $n - 1$, the system is not memoryless.

(*b*) Since the output does not depend on the future input values, the system is causal.

(*c*) Let $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$. Then

$$y[n] = \mathbf{T}\{\alpha_1 x_1[n] + \alpha_2 x_2[n]\} = \alpha_1 x_1[n-1] + \alpha_2 x_2[n-1]$$
$$= \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

Thus, the superposition property (1.68) is satisfied and the system is linear.

(*d*) Let $y_1[n]$ be the response to $x_1[n] = x[n - n_0]$. Then

$$y_1[n] = \mathbf{T}\{x_1[n]\} = x_1[n-1] = x[n-1-n_0]$$

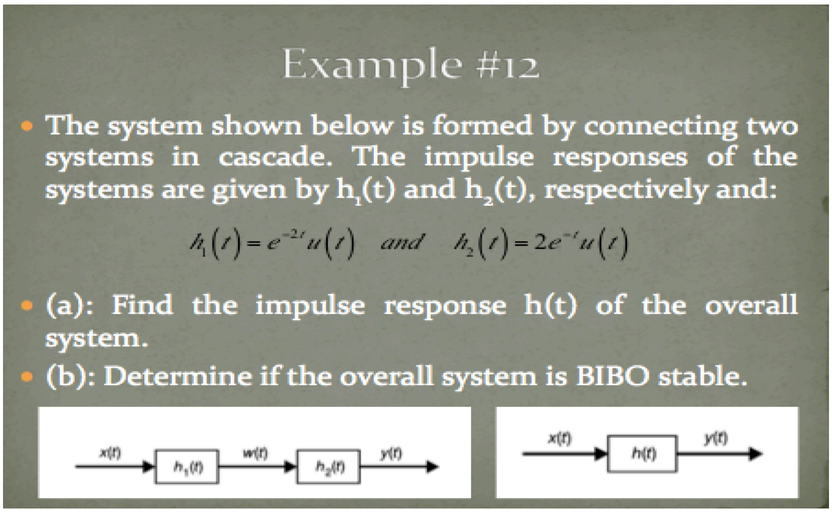
and $$y[n-n_0] = x[n-n_0-1] = x[n-1-n_0] = y_1[n]$$

Hence, the system is time-invariant.

(*e*) Since

$$|y[n]| = |x[n-1]| \le k \qquad \text{if } |x[n]| \le k \text{ for all } n$$

the system is BIBO stable.



(*a*) Let $w(t)$ be the output of the first system. By Eq. (2.6)

$$w(t) = x(t) * h_1(t)$$

Then we have

$$y(t) = w(t) * h_2(t) = [x(t) * h_1(t)] * h_2(t)$$

But by the associativity property of convolution (2.8), Eq. (2.79) can be rewritten as

$$y(t) = x(t) * [h_1(t) * h_2(t)] = x(t) * h(t)$$

Therefore, the impulse response of the overall system is given by

$$h(t) = h_1(t) * h_2(t)$$

Thus, with the given $h_1(t)$ and $h_2(t)$, we have

$$\begin{aligned} h(t) &= \int_{-\infty}^{\infty} h_1(\tau)h_2(t-\tau)\,d\tau = \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)\,2e^{-(t-\tau)}u(t-\tau)\,d\tau \\ &= 2e^{-t}\int_{-\infty}^{\infty} e^{-\tau}u(\tau)u(t-\tau)\,d\tau = 2e^{-t}\left[\int_0^t e^{-\tau}\,d\tau\right]u(t) \\ &= 2(e^{-t} - e^{-2t})u(t) \end{aligned}$$

(*b*) Using the above $h(t)$, we have

$$\begin{aligned} \int_{-\infty}^{\infty} |h(\tau)|\,d\tau &= 2\int_0^{\infty}(e^{-\tau} - e^{-2\tau})\,d\tau = 2\left[\int_0^{\infty} e^{-\tau}\,d\tau - \int_0^{\infty} e^{-2\tau}\,d\tau\right] \\ &= 2\left(1 - \frac{1}{2}\right) = 1 < \infty \end{aligned}$$

Thus, the system is BIBO stable.