day / date: ~~Thursday~~ Friday / 21 / ~~20~~ 18

LECTURE # 11

:- SOLVED EXAMPLES :-

EXAMPLE # 1 :-

$$x[n] = \delta[n] + \delta[n-1] + \delta[n+1]$$

SOLUTION :-

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \left(\delta[n] + \delta[n-1] + \delta[n+1] \right) e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \delta[n] e^{-jwn} + \sum_{n=-\infty}^{\infty} \delta[n-1] e^{-jwn} + \sum_{n=-\infty}^{\infty} \delta[n+1] e^{-jwn}$$

$$X(e^{jw}) = 1 + e^{-jw} + e^{jw}$$

$$\Rightarrow 1 + \left[2 \cos w \right]$$

EXAMPLE # 2 :-

a) $x[n] = x[1-n] + x[-1-n]$

Sol :-

Using the time reversal property, we have

$$x[-n] \overset{FT}{\longleftrightarrow} X(e^{-jw})$$

Using the shifting property, we have

$$x[-n+1] \overset{FT}{\longleftrightarrow} e^{-jwn} X(e^{-jw})$$

&

$$x[-1-n] \overset{FT}{\longleftrightarrow} e^{jwn} X(e^{-jw})$$

Therefore,

$$x[n] = x[1-n] + x[-1-n] \overset{FT}{\longleftrightarrow} e^{-jwn} X(e^{-jw}) + e^{jwn} X(e^{-jw})$$

$$\overset{FT}{\longleftrightarrow} X(e^{-jw}) \left[e^{-jwn} + e^{jwn} \right]$$

$$\overset{FT}{\longrightarrow} 2 X(e^{-jw}) \cos w.$$

b) $x(n) = (n-1)^2 x(n)$

Sol:

Using the differentiation in frequency property, we have

$$n x(n) \overset{FT}{\longleftrightarrow} j \frac{dX(e^{jw})}{dw}$$

Using the same property a second time,

$$n^2 x(n) \overset{FT}{\longleftrightarrow} -\frac{d^2 X(e^{jw})}{dw^2}$$

Therefore,

$$x(n) = n^2 x(n) - 2n x(n) + 1 \overset{FT}{\longleftrightarrow} -\frac{d^2 X(e^{jw})}{dw^2} - 2j\frac{dX(e^{jw})}{dw} + X(e^{jw}).$$

EXAMPLE #3:

$$y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = 2x(n)$$

$$x(n) = \left(\frac{1}{4}\right)^n u(n)$$

$y(n) = ?$

Sol:

$$Y(e^{jw}) - \frac{3}{4} Y(e^{+jw}) e^{-jw} + \frac{1}{8} e^{-2jw} Y(e^{jw}) = 2X(e^{jw})$$

$$Y(e^{jw}) \left(1 - \frac{3}{4} e^{-jw} + \frac{1}{8} e^{-j2w}\right) = 2X(e^{jw})$$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{2}{1 - \frac{3}{4} e^{-jw} + \frac{1}{8} e^{-j2w}}$$

Factor the denominator of $H(e^{jw})$ i-e;

$$1 - \frac{3}{4} e^{-jw} + \frac{1}{8} e^{-j2w} = \left(1 - \frac{1}{2} e^{-jw}\right)\left(1 - \frac{1}{4} e^{-jw}\right)$$

$$H(e^{jw}) = \frac{2}{\left[1-\frac{1}{2}e^{-jw}\right]\left[1-\frac{1}{4}e^{-jw}\right]}$$

Now,

$$x[n] = \left[\frac{1}{4}\right]^n u[n] \qquad \because a^n u[n] = \frac{1}{1-ae^{-jw}}$$

$$X(e^{jw}) = \frac{1}{1-\frac{1}{4}e^{-jw}}$$

$$Y(e^{jw}) = H(e^{jw})\, X(e^{jw})$$

$$= \left[\frac{2}{\left[1-\frac{1}{2}e^{-jw}\right]\left[1-\frac{1}{4}e^{-jw}\right]}\right]\left(\frac{1}{1-\frac{1}{4}e^{-jw}}\right)$$

$$Y(e^{jw}) = \frac{2}{\left[1-\frac{1}{2}e^{-jw}\right]\left[1-\frac{1}{4}e^{-jw}\right]^2}$$

Using partial fraction method :

$$Y(e^{jw}) = \frac{2}{\left[1-\frac{1}{2}e^{-jw}\right]\left[1-\frac{1}{4}e^{-jw}\right]^2} = \frac{A}{\left[1-\frac{1}{4}e^{-jw}\right]} + \frac{B}{\left[1-\frac{1}{4}e^{-jw}\right]^2} + \frac{C}{1-\frac{1}{2}e^{-jw}}$$

Cross multiplication gives:

$$2 = A\left[1-\frac{1}{2}e^{-jw}\right]\left[1-\frac{1}{4}e^{-jw}\right] + B\left[1-\frac{1}{2}e^{-jw}\right] + C\left[1-\frac{1}{4}e^{-jw}\right]^2$$

let $e^{-jw} = s$

$$2 = A\left[1-\frac{1}{2}s\right]\left[1-\frac{1}{4}s\right] + B\left[1-\frac{1}{2}s\right] + C\left[1-\frac{1}{4}s\right]^2$$

let $s=2$, $s=4$

when $s=2$,

$$2 = A\left[1-\frac{1}{\cancel{2}}(\cancel{2})\right]\left[1-\frac{1}{\cancel{4}_2}(\cancel{2})\right] + B\left[1-\frac{1}{\cancel{2}}(\cancel{2})\right] + C\left[1-\frac{1}{\cancel{4}_2}(\cancel{2})\right]^2$$

$$2 = A(0)\left(1-\frac{1}{2}\right) + B(0) + C\left[1-\frac{1}{2}\right]^2$$

$$2 = C\left[\frac{2-1}{2}\right]^2 \Rightarrow 2 = C\left[\frac{1}{4}\right]$$

$$C \Rightarrow 8$$

when $s=4$,

$$2 = A\left[1-\frac{1}{\cancel{2}_1}(\cancel{4}^2)\right]\left[1-\frac{1}{\cancel{4}}(\cancel{4})\right] + B\left[1-\frac{1}{\cancel{2}_1}(\cancel{4}^2)\right] + C\left[1-\frac{1}{\cancel{4}}(\cancel{4})\right]^2$$

$$2 = A(0) + B[1-2] + C[0]$$

$$B \Rightarrow -2$$

For the value of A we will put $s=0$ as we don't have any other value for s from the equation above.

Hence, when $s=0$

$$2 = A\left[1-\frac{1}{2}(0)\right]\left[1-\frac{1}{4}(0)\right] + B\left[1-\frac{1}{2}(0)\right] + C\left[1-\frac{1}{4}(0)\right]^2$$

$$2 = A[1][1] + (-2)(1) + (8)(1)$$

$$2 = A - 2 + 8$$

$$2 = A + 6$$

$$A = -6 + 2 \Rightarrow -4$$

So, $$Y(e^{jw}) = \frac{-4}{1-\frac{1}{4}e^{-jw}} - \frac{2}{\left(1-\frac{1}{4}e^{-jw}\right)^2} + \frac{8}{1-\frac{1}{2}e^{-jw}}$$

Using the properties :-

$$a^n u[n] = \frac{1}{1-ae^{-jw}}$$

$$\frac{1}{(1-ae^{-jw})^2} = (n+1)a^n u[n]$$

The inverse fourier transform $y[n]$ is :-

$$y[n] = \left[-4\left(\frac{1}{4}\right)^n - 2(n+1)\left(\frac{1}{4}\right)^n + 8\left(\frac{1}{2}\right)^n\right] u[n]$$