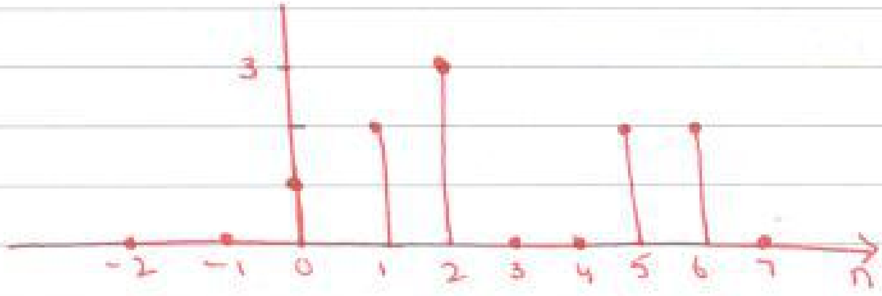
-: LECTURE # 6 :-
-: REVISION :-

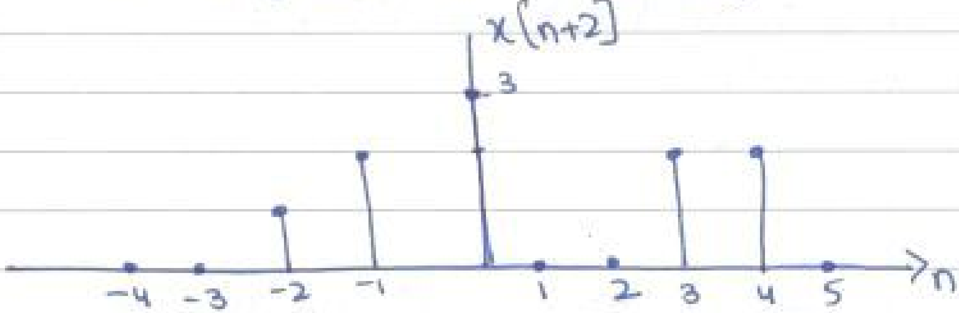
Example # 1 :-



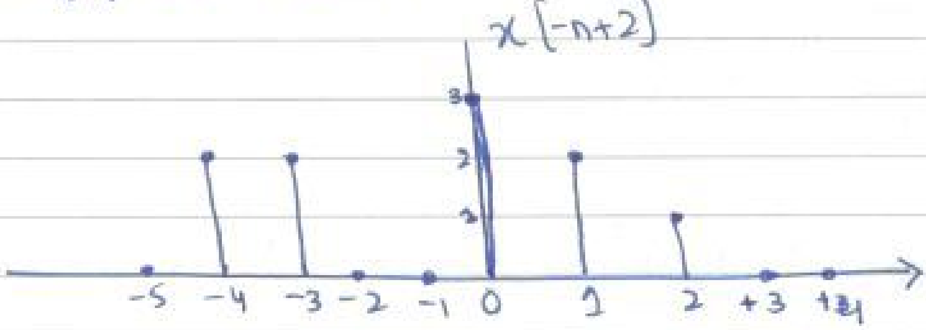
$4x[-n+2]$

Sol :-

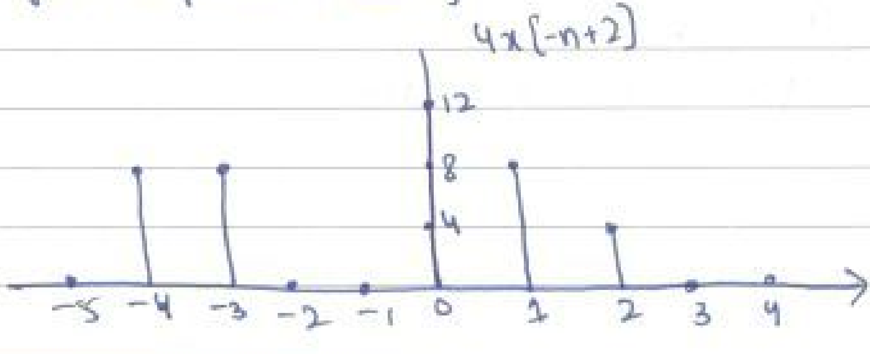
First shift 2 points towards left



Now flip over x-axis.



Now perform amplitude scaling.



EXAMPLE #2:-

1) $x(t) = 0.9e^{-3t}\,u(t)$

Sol:-

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$= \int_{-\infty}^{\infty} |0.9e^{-3t}u(t)|^2\,dt \quad = 0.81\int_0^{\infty} e^{-6t}\,dt$$

$$= 0.81\left[\frac{e^{-6t}}{-6}\right]_0^{\infty} \quad = 0.81\left[\frac{-e^{-\infty}}{6} + \frac{e^0}{6}\right]$$

$$= 0.81\left[\frac{1}{6}\right] \Rightarrow E = 0.135\,J$$

Since the signal is not periodic hence it's not a power signal but it is a energy signal.

2) $x[n] = u[n]$

Sol:-

The signal is periodic because $u[n]$ repeats after every sample and of infinite duration. Hence it is a Power Signal.

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |u[n]|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} (1)^2$$

Since $\sum_{0}^{N} (1)^2 = 1 + 1 + 1 + \dots \infty = (N+1)$

$$P = \lim_{N \to \infty} \frac{1}{2N+1}(N+1)$$

$$= \lim_{N \to \infty} \frac{\cancel{N}\left(1+\frac{1}{N}\right)}{\cancel{N}\left(2+\frac{1}{N}\right)} \Rightarrow 0.5W$$

$$P = 0.5W$$

Example #3:-

1) $x(t) = \cos(2\pi t) + \sin(10\pi t)$

Sol:-

$$x_1(t) = \cos(2\pi t) \quad , \quad x_2(t) = \sin(10\pi t)$$

$$\omega_1 = 2\pi \quad , \quad \omega_2 = 10\pi$$

Step:1 $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi} \Rightarrow 1 \quad , \quad T_2 = \frac{\cancel{2\pi}}{\cancel{10\pi}} \Rightarrow \frac{1}{5}$

Step:2 $\frac{T_1}{T_2} = \frac{1}{1/5} = \frac{1}{1} \times 5 \Rightarrow 5$ rational.

Step:3 $T_0 = LCM(T_1, T_2)$

$= LCM(1, 1/5)$

$= \frac{\text{LCM of Numerator}}{\text{Denominator of HCF}} = \frac{1}{1}$

$T_0 = \frac{1}{1} \Rightarrow 1$

2) $x[n] = 3\sin(3\pi n + \pi/2)$

Sol:

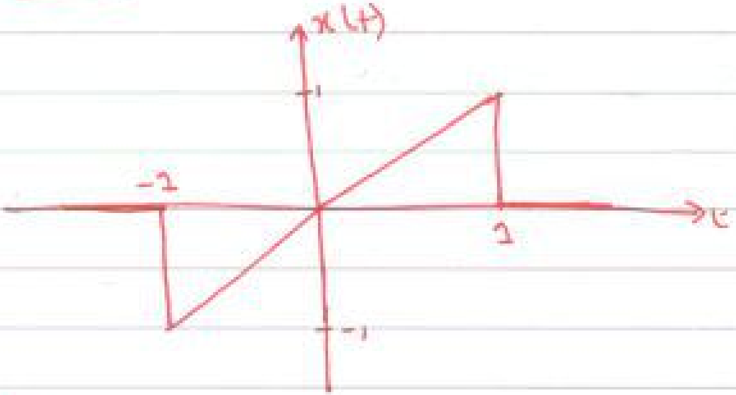
$$\omega = 3\pi$$

$$\frac{N}{m} = \frac{2\cancel{\pi}}{3\cancel{\pi}} \quad , \quad m = 3$$

$$N = 2$$

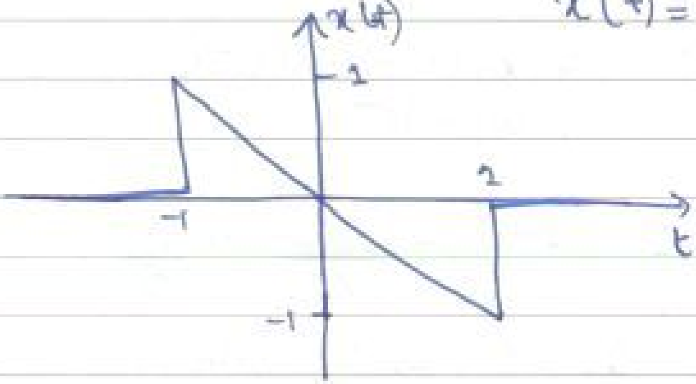
Example #4:-

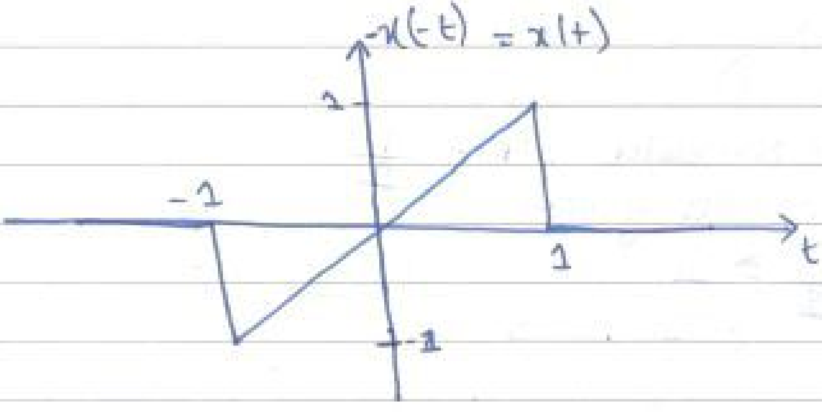
1)



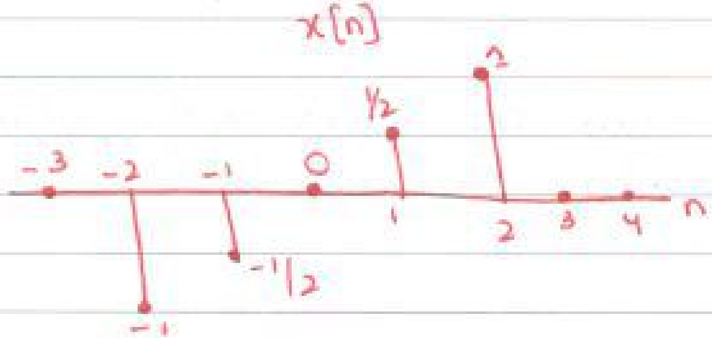
Sol:-

$x(t) = -x(-t)$, $x(t)$ is odd in this case.



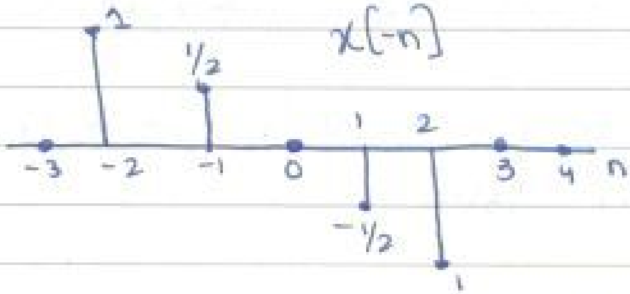


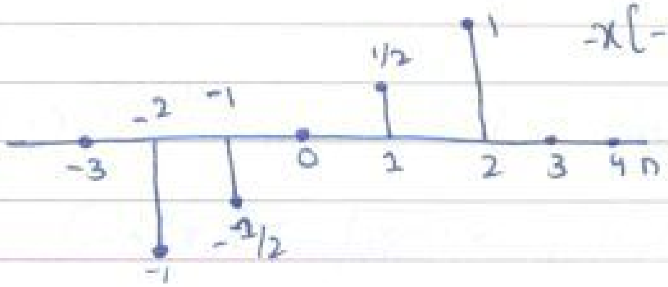
2)



Sol:

$x[n] = -x[-n]$, $x[n]$ is odd.





Example #5:

$x[n] \longrightarrow \boxed{\text{Unit Delay}} \longrightarrow y[n] = x[n-1]$

1) Memoryless:

The system input-output relation is given by.

$$y[n] = x[n-1]$$

Since the output value at n depends on the input values at n-1, the system is not memoryless.

2) Causal :-

Since the output does not depend on the future input values, the system is causal.

3) Linear :-

Law of Addition :-

$$y_1[n] = x_1[n-1] \quad , y_2[n] = x_2[n-1]$$

$$y'[n] = y_1[n] + y_2[n] = x_1[n-1] + x_2[n-1]$$

$$x_1[n] + x_2[n] = y_3[n] = x_1[n-1] + x_2[n-1]$$

$$\therefore y'[n] = y_3[n]$$

Law of Homogenity :-

$$x[n] \rightarrow \boxed{\text{unit delay}} \rightarrow y[n] = x[n-1] \xrightarrow{'k'} ky[n] = kx[n-1]$$

$$x[n] \xrightarrow{'k'} kx[n] \rightarrow \boxed{\text{unit delay}} \rightarrow kx[n-1]$$

Hence both the laws are satisfied, the system is linear.

4) Time invariant :-

$$x[n] \rightarrow \boxed{\text{unit delay}} \rightarrow y[n] = x[n-1] \xrightarrow{n-n_0} y[n-n_0] = x[n-n_0-1]$$

$$x[n] \xrightarrow{n_0} x[n-n_0] \longrightarrow \boxed{\text{unit delay}} \longrightarrow y'[n-n_0] = x[n-n_0-1]$$

Hence both are same the system is time invariant.

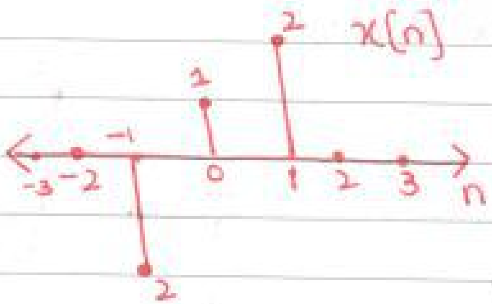
5) Stable:

Since

$$|y[n]| = |x[n-1]| \leq K \quad \text{if } |x[n]| \leq K \text{ for all } n$$

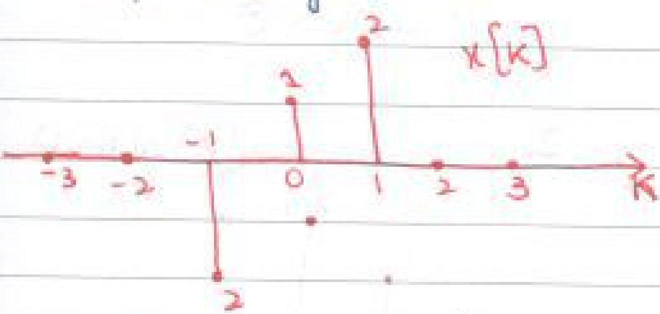
The system is BIBO stable.

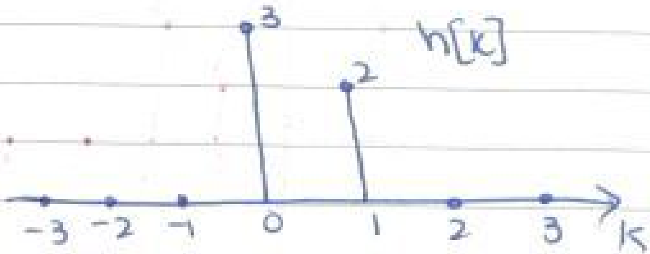
EXAMPLE #6:



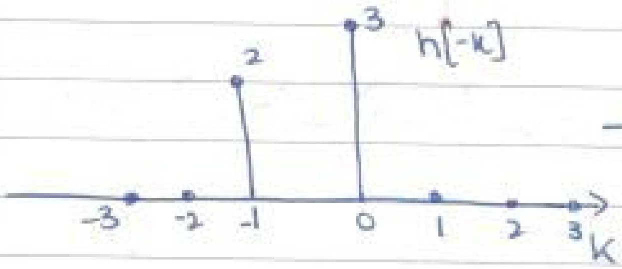
Sol:

Step 1 Change $n \to k$

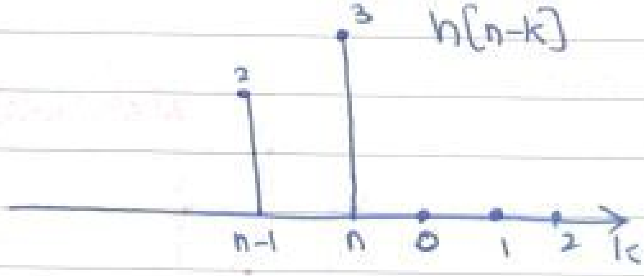




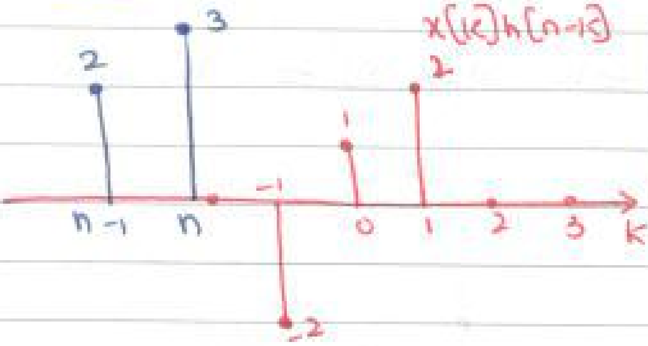
Step 2 Flip and shift $h[k]$



$\rightarrow$



Step 3 : Now convolve $x[k]$ and $h[n-k]$

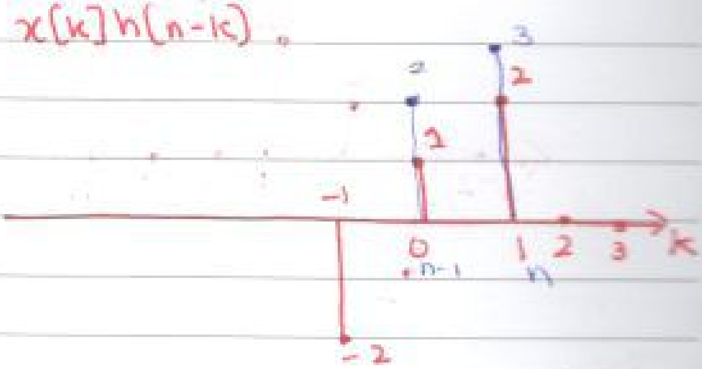


$y[0] = [1\times3] + [-2\times2]$

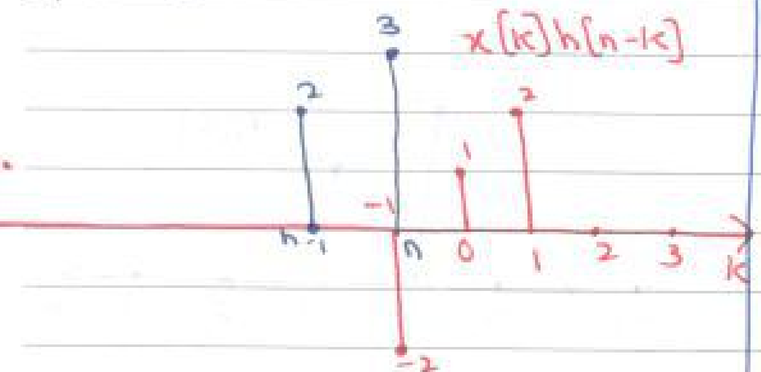
$= [3] - [4] \Rightarrow -1$

when $n < -1$ no overlapping
$y[n] = 0$

$\therefore n = 1$



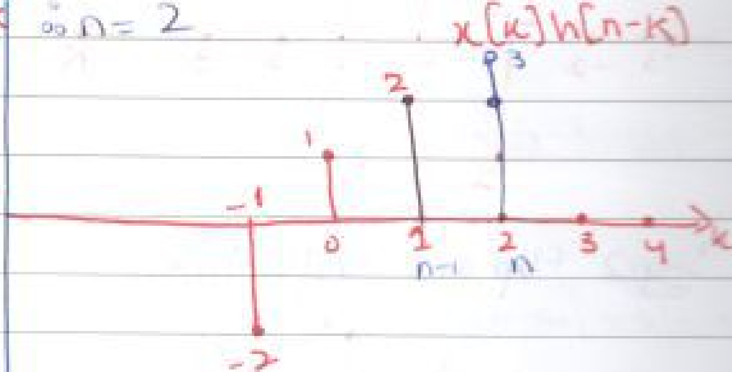
$\therefore n = -1$



$y[1] = [2\times3] + [1\times2]$

$= 6 + 2 \Rightarrow 8$

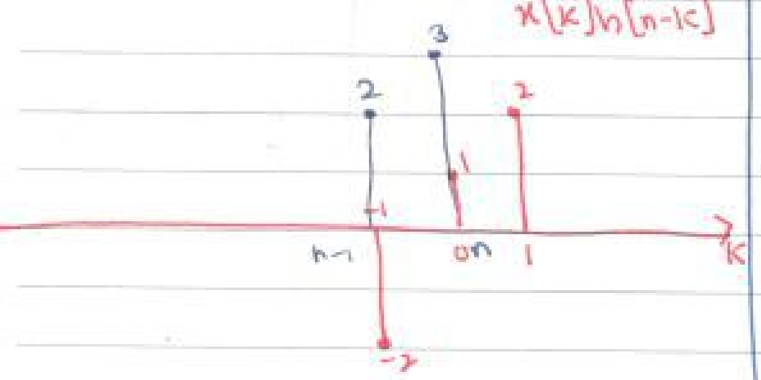
$\therefore n = 2$



$y[-1] = [-2\times3] + (0\times2)$

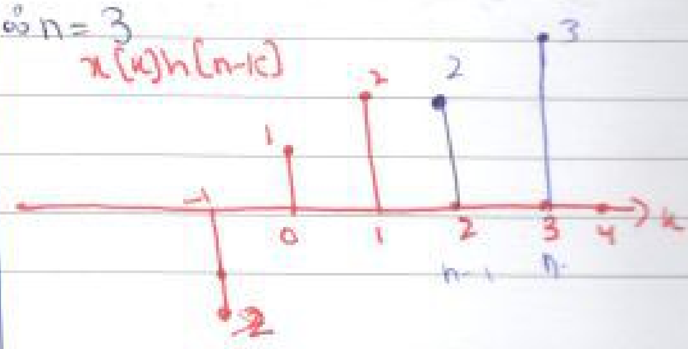
$= -6 + 0 \Rightarrow -6$

$\therefore n = 0$



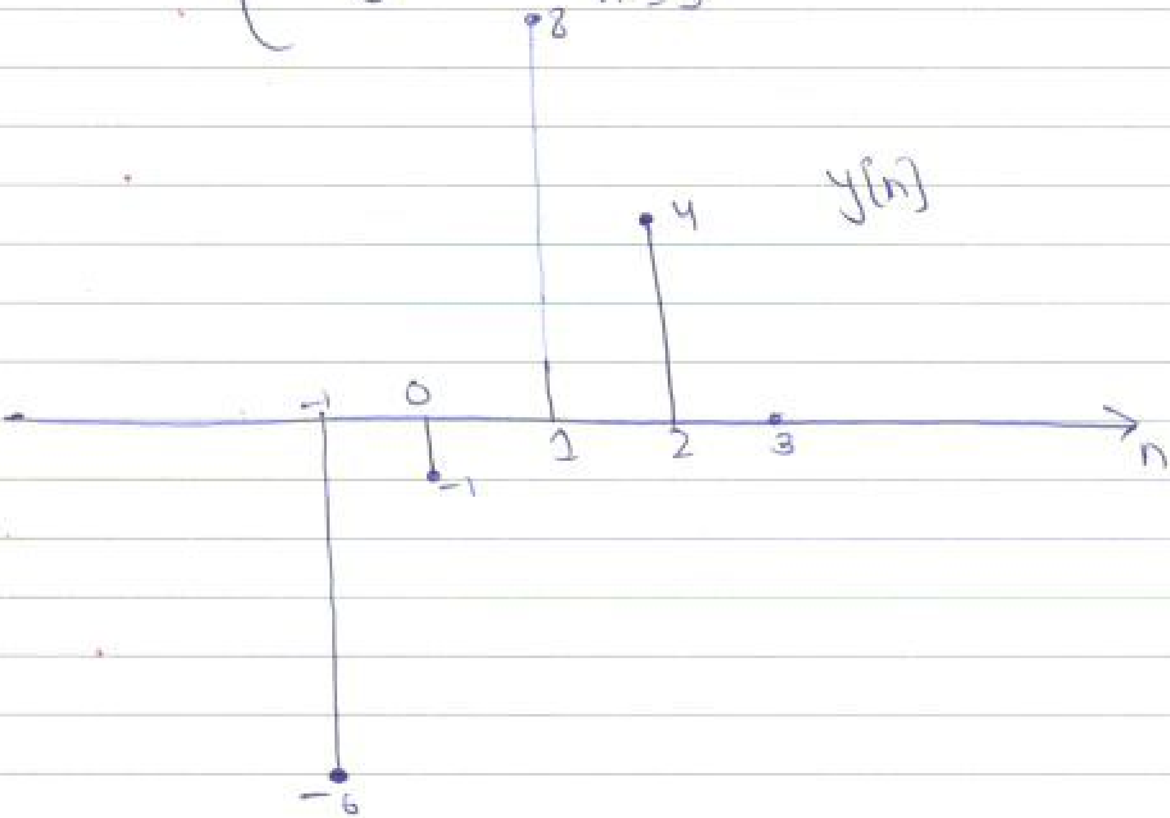
$y[2] = (0\times3) + (2\times2) \Rightarrow 4$

$\therefore n = 3$

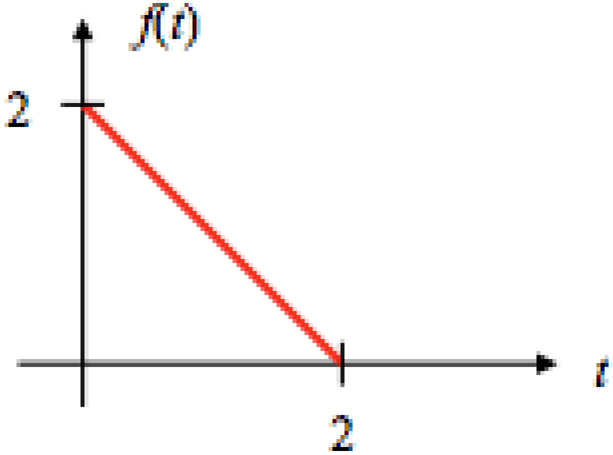


$y[3] =$ no overlapping $= 0$

Step#4- $y[n] = \begin{cases} 0 & n < -1 \\ -6 & n = -1 \\ -1 & n = 0 \\ 8 & n = 1 \\ 4 & n = 2 \\ 0 & n = 3 \end{cases}$



- **Convolve the following two functions:**



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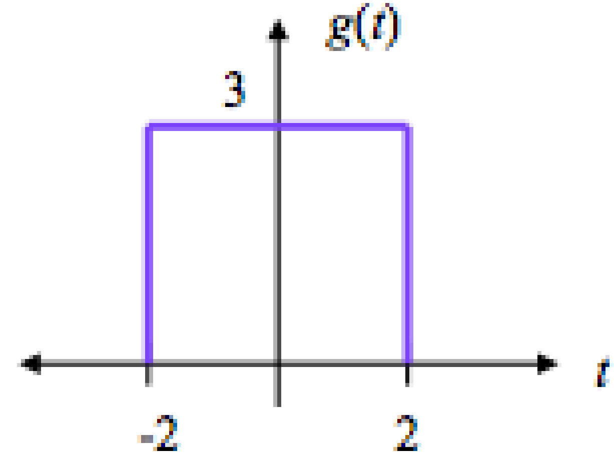
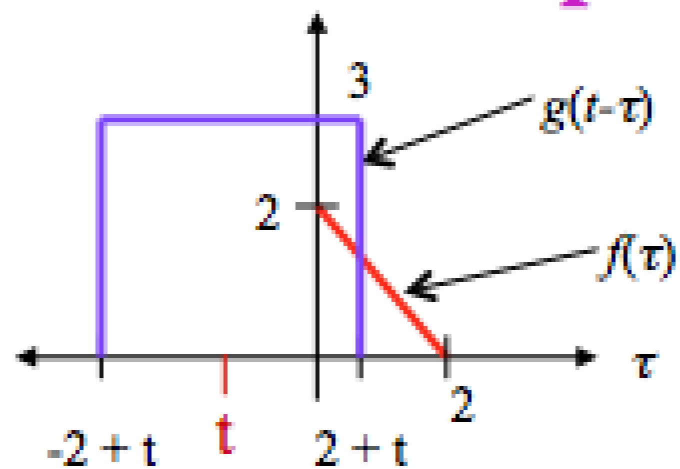


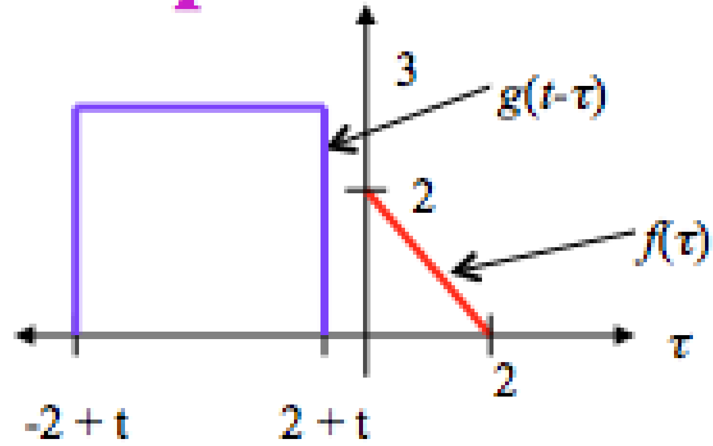
- **Replace t with τ in $f(t)$ and $g(t)$**
- **Choose to flip and slide $g(\tau)$ since it is simpler and symmetric**
- **Functions overlap like this:**



- **Convolution can be divided into 5 parts**

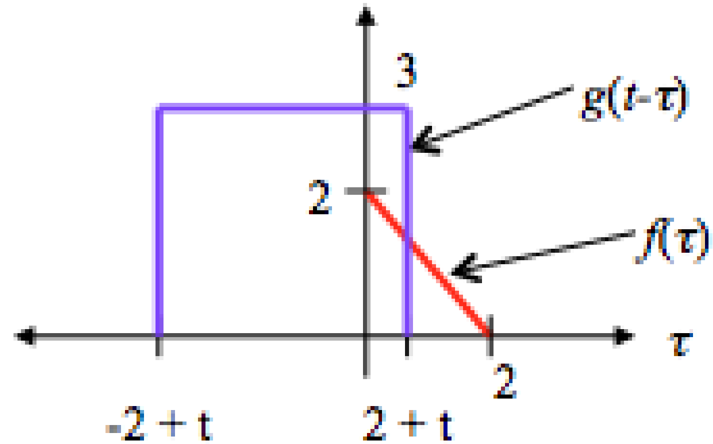
I. $t < -2$

- **Two functions do not overlap**
- **Area under the product of the functions is zero**



II. $-2 \le t < 0$

- **Part of $g(t)$ overlaps part of $f(t)$**
- **Area under the product of the functions is**

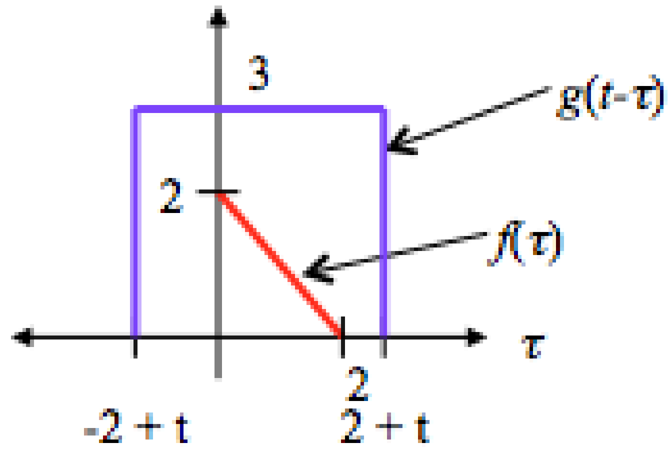


$$\int_0^{2+t} 3(-\tau+2)\,d\tau = 3\left(-\frac{\tau^2}{2}+2\tau\right)\Bigg|_0^{2+t} = -\frac{3(2+t)^2}{2}+6(2+t) = -\frac{3t^2}{2}+6$$

III. $0 \le t < 2$

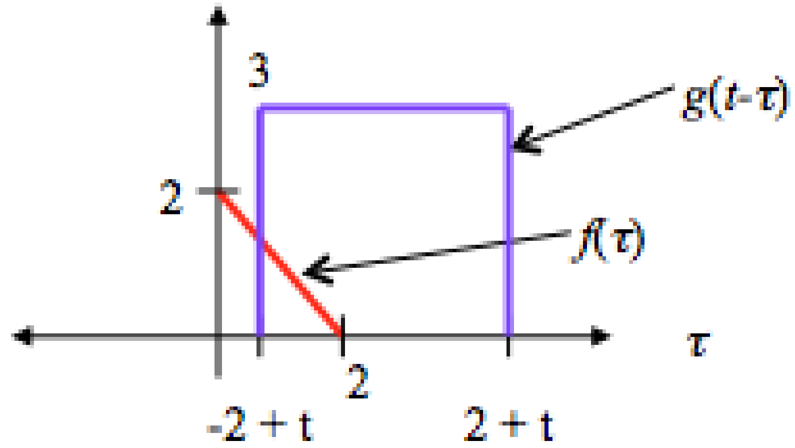
- **Here, $g(t)$ completely overlaps $f(t)$**
- **Area under the product is just**

$$\int_0^2 3(-\tau + 2)\,d\tau = 3\left(-\frac{\tau^2}{2} + 2\tau\right)\Bigg|_0^2 = 6$$



V. $2 \le t < 4$

- **Part of $g(t)$ and $f(t)$ overlap**
- **Calculated similarly to $-2 \le t < 0$**



VII. $t \ge 4$

- **$g(t)$ and $f(t)$ do not overlap**
- **Area under their product is zero**

- **Result of convolution (5 intervals of interest):**

$$y(t) = f(t) * g(t) = \begin{cases} 0 & \text{for } t < -2 \\ -\dfrac{3}{2}t^2 + 6 & \text{for } -2 \le t < 0 \\ 6 & \text{for } 0 \le t < 2 \\ \dfrac{3}{2}t^2 - 12t + 24 & \text{for } 2 \le t < 4 \\ 0 & \text{for } t \ge 4 \end{cases}$$

No Overlap

Partial Overlap

Complete Overlap

Partial Overlap

No Overlap