LECTURE #9

EXAMPLE #1:-

1) $y(t) = x(t+1) + x(t-1)$

SOL:-

$$x(t) \rightleftharpoons C_n$$
$$x(t \pm t_0) \rightleftharpoons C_n e^{\pm jnw_0 t_0}$$
$$x(t+1) \rightleftharpoons C_n e^{+jnw_0 1}$$
$$x(t-1) \rightleftharpoons C_n e^{-jnw_0 1}$$
$$y(t) = x(t+1) + x(t-1)$$
$$= C_n e^{jnw_0} + C_n e^{-jnw_0}$$
$$= C_n \left[e^{jnw_0} + e^{-jnw_0}\right]$$
$$y(t) = 2C_n \cos nw_0 = C'_n$$

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2) $y(t) = e^{-j2w_0 t} \cdot x(t)$

SOL:-

$$\because e^{jmw_0 t} \cdot x(t) \rightleftharpoons C_{n-m}$$
$$y(t) = e^{-j2w_0 t} \cdot x(t) \rightleftharpoons C'_n$$

$$e^{j\underset{m}{\underbrace{(-2)}} w_0 t} \cdot x(t) \rightleftharpoons C'_n = C_{n-(-2)}$$
$$C'_n = C_{n+2}$$

EXAMPLE #2:-

$$\frac{dy(t)}{dt} + 4y(t) = x(t)$$

$C'_n = ?$ $\quad$ i/p $\Rightarrow \cos 2\pi t + \sin 4\pi t$

SOL:- $\because C'_n = H(jnw_0) \cdot C_n$

$$x(t) = \cos 2\pi t + \sin 4\pi t$$
$$w_{0_1} = 2\pi \quad , \quad w_{0_2} = 4\pi$$

$$\omega_0 = HCF(\omega_{0_1}, \omega_{0_2})$$
$$= HCF(2\pi, 4\pi)$$
$$\omega_0 = 2\pi$$

$$x(t) = \cos 2\pi t + \sin 4\pi t$$
$$= \cos \omega_0 t + \sin 2\omega_0 t$$
$$= \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} + \frac{1}{2j}e^{j2\omega_0 t} - \frac{1}{2j}e^{-j2\omega_0 t}$$

$$C_1 = \frac{1}{2}, \quad C_{-1} = \frac{1}{2}$$

$$C_2 = \frac{1}{2j}, \quad C_{-2} = \frac{-1}{2j}$$

$$C_1' = H(jn\omega_0) \times C_1$$
$$= H(j1\underset{2\pi}{\omega_0}) \times C_1$$

$$C_1' = \frac{1}{4+j2\pi} \times \frac{1}{2}$$

$$C_{-1}' = H(-j\underset{2\pi}{\omega_0}) \times C_{-1}$$
$$= \frac{1}{4-j2\pi} \times \frac{1}{2}$$

$$C_2' = H(j2\underset{2\pi}{\omega_0}) \times C_2$$
$$= \frac{1}{4+j4\pi} \times \frac{1}{2j}$$

$$C_2' = \underbrace{\frac{1}{4-j4\pi}}_{H(-j2\omega_0)} \times \underbrace{(-1/2j)}_{C_2}$$

$$\frac{dy(t)}{dt} + 4y(t) = x(t)$$
$$s.Y(s) + 4Y(s) = X(s) \Rightarrow Y(s)[s+4] = X(s)$$
$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{4+s}$$

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EXAMPLE #3:-

$x(n)$ = real and odd periodic signal.

$N=7, \quad a_k = ?$

$a_{15} = j, \quad a_{16} = 2j, \quad a_{17} = 3j$

$a_0 = ? \quad a_{-1} = a_{-2} = a_{-3} = ?$

Sol:-

Since the Fourie series coefficient repeat every N, we have

$a_1 = a_{15}, \quad a_2 = a_{16}, \quad a_3 = a_{17}$

Furthermore since the signal is real and odd, the Fourier series coefficients a_k will be purely imaginary and odd. Therefore $a_0 = 0$ and.

$a_1 = -a_{-1}, \quad a_2 = -a_{-2}, \quad a_3 = -a_{-3}$

Finally,

$a_{-1} = -j, \quad a_{-2} = -2j, \quad a_{-3} = -3j.$