ISRA UNIVERSITY

Islamabad Campus

Program: BSC & MSC (Electrical)

Semester - Fall 2018

Solution

Signal & Systems

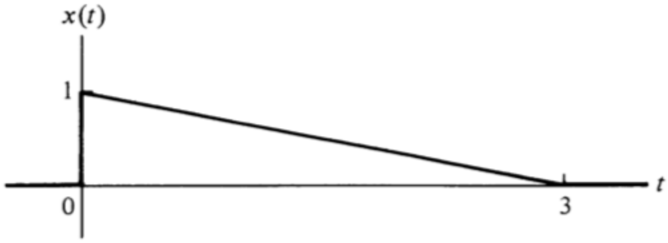
Assignment – 4 | **Due Date: 17/01/2019**

Marks: 30 | **Handout Date: 02/01/2019**

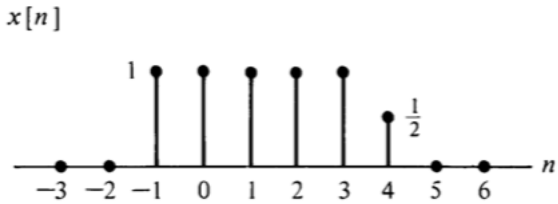
Question # 1:

Sketch and label each of the following signals:

1. $x(2t+4)$



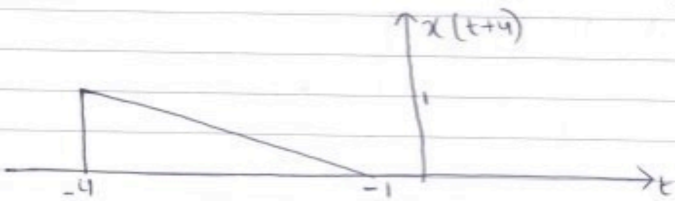
2. $-3x[n+2]$



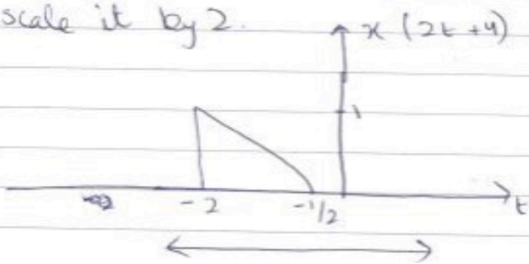
Solution:

Sol:

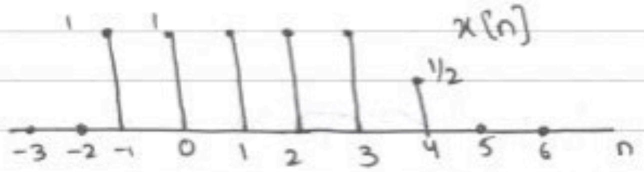
First shift x(t) 4 points towards left.



Now scale it by 2.

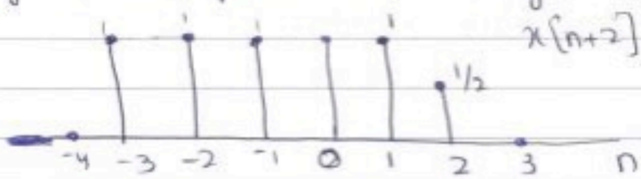


2) $-3x[n+2]$

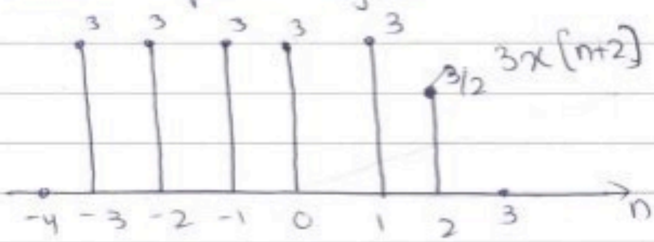


Sol:

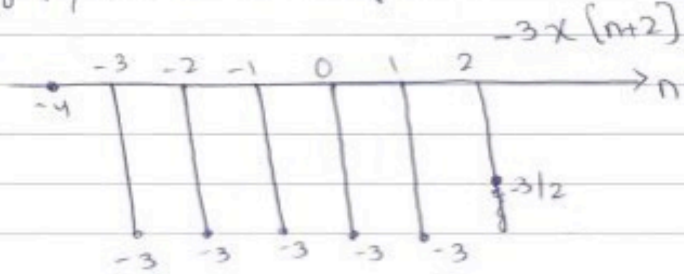
Shift $x[n]$ 2 points towards left.



Now scale amplitude by 3.



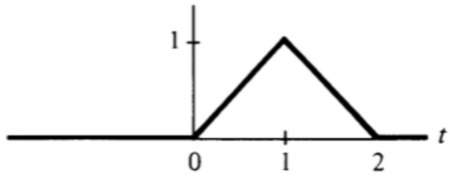
Now flip/reverse the amplitude.

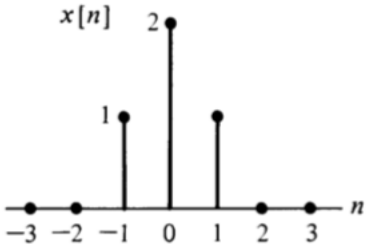


Question # 2:

For each of the following signals, determine whether it is even, odd or neither:

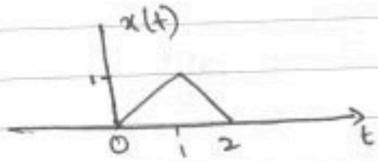
1.



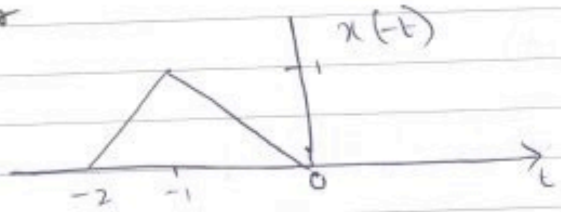
2. 

Solution:

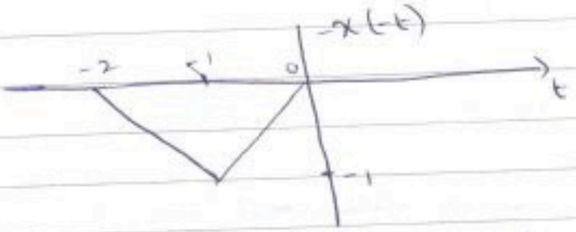
Q#2:

1) 

Sol:

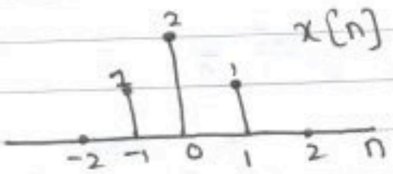


$$x(t) \neq x(-t)$$

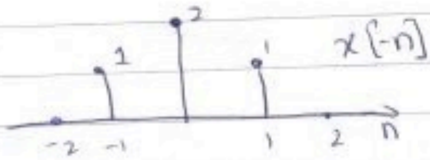


$$x(t) \neq -x(-t)$$

Thus $x(t)$ is neither even nor odd.

2) 

Sol:



$$x[n] = x[-n]$$

Hence, $x[n]$ is an even signal.

Question # 3:

Consider the signals:

$$x(t) = \cos\frac{2\pi t}{3} + 2\sin\frac{16\pi t}{3}$$

$$y(t) = \sin \pi t$$

Show that z (t)= x (t) y (t) is periodic. Find the fundamental period of z (t).

Solution:

Sol:

$$z(t) = x(t)\,y(t)$$

$$= (\sin \pi t)\left(\cos\frac{2\pi t}{3} + 2\sin\frac{16\pi t}{3}\right)$$

Decompose it into sum of exponentials using Euler's identity

$$\sin(\pi t) = \frac{e^{jt\pi} - e^{-j\pi t}}{2j}$$

$$\cos\frac{2\pi}{3}t = \frac{e^{j2\pi/3 t} + e^{-j\frac{2\pi}{3}t}}{2}$$

$$\sin\frac{16\pi t}{3} = \frac{e^{j\frac{16\pi t}{3}} - e^{-j\frac{16\pi}{3}t}}{2j}$$

$$z(t) = \left[\frac{e^{jt\pi} - e^{-j\pi t}}{2j}\right]\left[\frac{e^{j\frac{2\pi}{3}t} + e^{-j\frac{2\pi}{3}t}}{2} + \cancel{2}\,\frac{e^{j\frac{16\pi t}{3}} - e^{-j\frac{16\pi}{3}t}}{\cancel{2}j}\right]$$

$$= \frac{e^{j\pi t}}{2j}\left[\frac{e^{j\frac{2\pi}{3}t}}{2} + \frac{e^{-j\frac{2\pi}{3}t}}{2} + \frac{\cancel{2}\,e^{j\frac{16\pi t}{3}}}{\cancel{2}j} - \frac{2\,e^{-j\frac{16\pi}{3}t}}{2j}\right]$$

$$- \frac{e^{-j\pi t}}{2j}\left[\frac{e^{j\frac{2\pi}{3}t}}{2} + \frac{e^{-j\frac{2\pi}{3}t}}{2} + \frac{e^{j\frac{16\pi}{3}}}{j} - \frac{e^{-j\frac{16\pi}{3}t}}{j}\right]$$

$$= \left[\frac{e^{j\pi t + \frac{2\pi}{3}t}}{4j} + \frac{e^{j\pi t - j\frac{2\pi}{3}t}}{4j} + \frac{e^{j\pi t + j\frac{16\pi t}{3}}}{2j^2} - \frac{e^{j\pi t - j\frac{16\pi}{3}t}}{2j^2}\right] - \left[\frac{e^{-j\pi t + \frac{2\pi}{3}t}}{4j}\right.$$

$$\left. - \frac{e^{-j\pi t - j\frac{2\pi}{3}t}}{4j} - \frac{e^{-j\pi t + j\frac{16\pi t}{3}}}{2j^2} + \frac{e^{-j\pi t - j\frac{16\pi}{3}t}}{2j^2}\right]$$

$$z(t) = \frac{1}{4j} e^{j(5\pi/3)t} - \frac{1}{4j} e^{-j(\pi/3)t} + \frac{1}{4j} e^{j(\pi/3)t} - \frac{1}{4j} e^{-j(5\pi/3)t}$$

$$- \frac{1}{2} e^{j(19\pi/3)t} + \frac{1}{2} e^{j(13\pi/3)t} + \frac{1}{2} e^{-j(13\pi/3)t} - \frac{1}{2} e^{-j(19\pi/3)t}$$

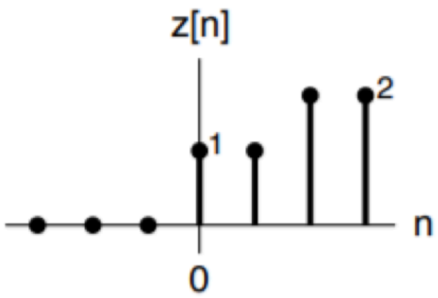
all exponential are powers of $e^{j(\pi/3)t}$.

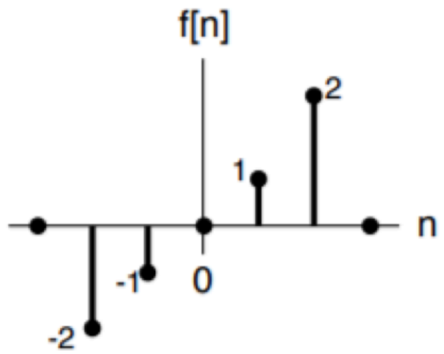
Thus, the fundamental period is $= \frac{2\pi}{\pi/3} = 2 \times 3 \Rightarrow 6$.

$z(t) = T_0 = 6$.

Question # 4:

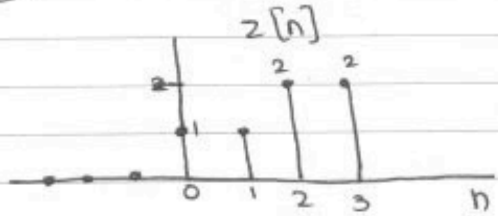
Determine the Discrete-time Convolution for the following signals:

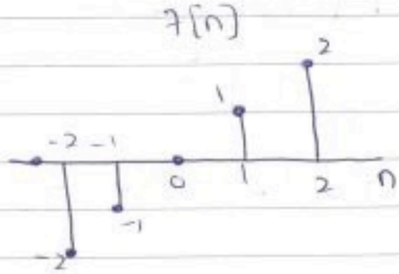




Solution:

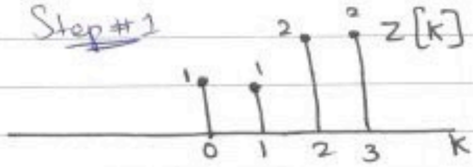
Q#4:-

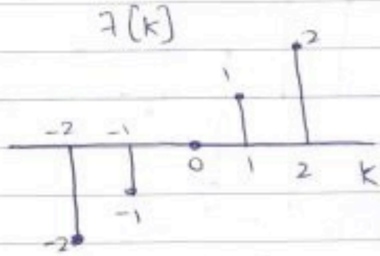




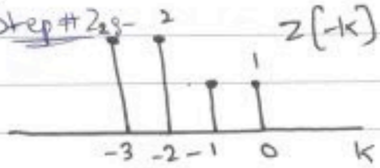
Sol:-

Step#1

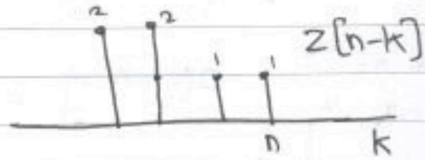




Step#2:-

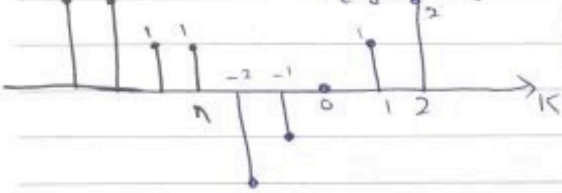


$\Rightarrow$



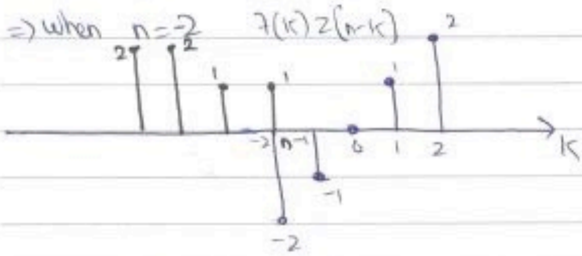
Step #3 Start overlapping.

$x[k]\, z[n-k]$



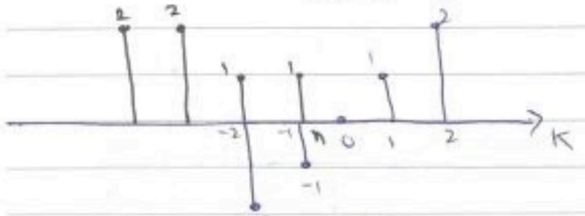
when $n < -2$ $y[n] = 0$ as there will be no overlapping.

$\Rightarrow$ when $n = -2$ $x(k)\, z(n-k)$



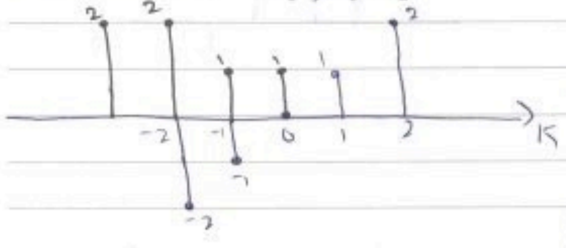
$$y[-2] = (-2 \times 1) \Rightarrow -2$$

$\Rightarrow$ when $n = -1$ $x[k]\, z[n-k]$



$$y[-1] = (-2 \times 1) + (-1 \times 1)$$
$$= -2 - 1 \Rightarrow -3$$

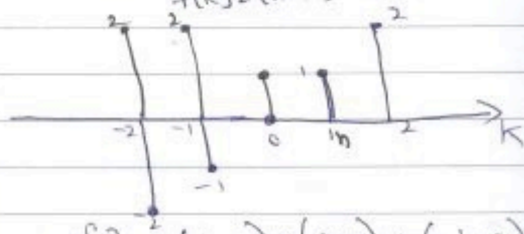
when $n = 0$ $x[k]\, z[n-k]$



$$y[0] = (0 \times 1) + (-1 \times 1) + (-2 \times 2)$$
$$= 0 - 1 - 4 \Rightarrow -5$$

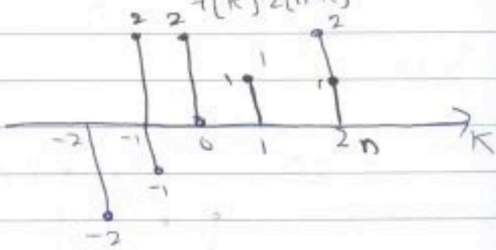
when $n = 1$

$x[k]\, z[n-k]$



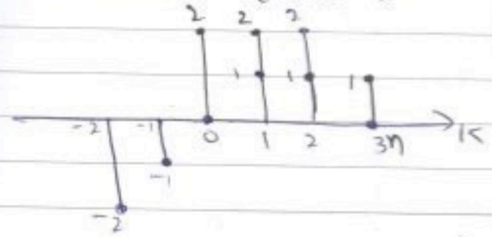
$$y[1] = (1 \times 1) + (0 \times 1) + (-1 \times 2) + (-2 \times 2)$$
$$= 1 + 0 - 2 - 4 \Rightarrow -5$$

when $n = 2$ $x[k]\, z[n-k]$



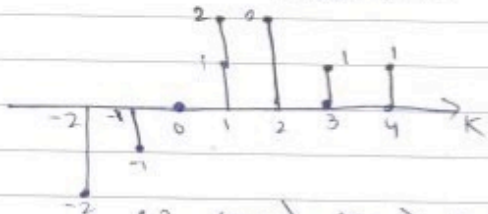
$$y[2] = (-1 \times 2) + (0 \times 2) + (1 \times 1) + (1 \times 2)$$
$$= -2 + 0 + 1 + 2 \Rightarrow 1$$

when $n=3$



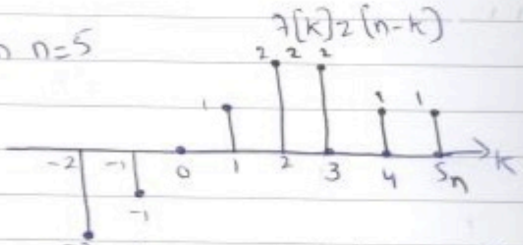
$$y[3] = (0\times2) + (1\times2) + (2\times1) + (0\times1)$$
$$= 0+2+2+0 = 4$$

when $n=4$



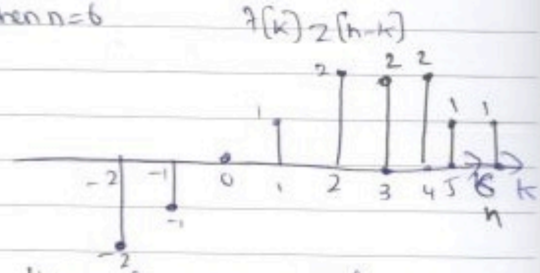
$$y[4] = (2\times2) + (2\times2) + (0\times1) + (0\times1)$$
$$= 2+4+0+0 \Rightarrow 6$$

when $n=5$



$$y[5] = (2\times2) + (2\times2) + (0\times1) + (0\times1)$$
$$= 4+4+0+0 \Rightarrow 8$$

when $n=6$

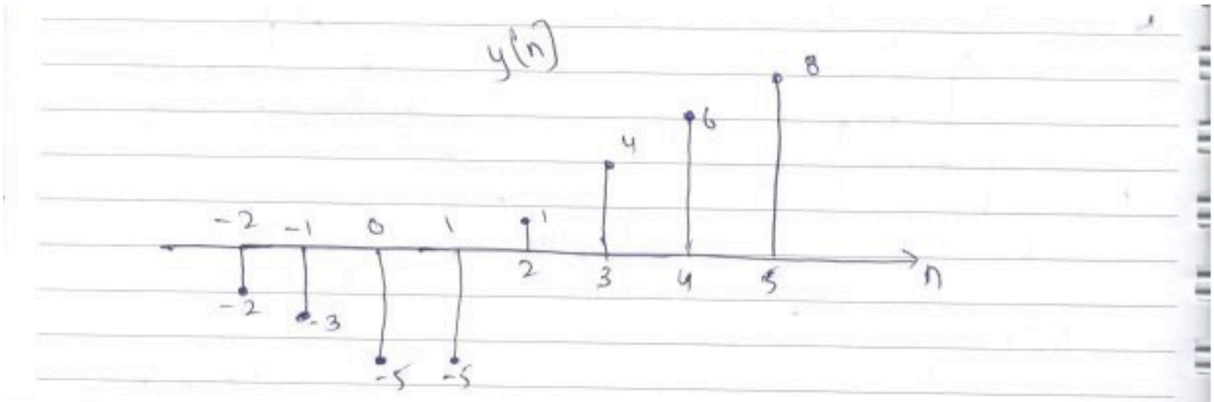


Hence, there is no overlapping

$\therefore$ $y[6] = 0$.

Step #4

$$y[n] = \begin{cases} 0 & n < -2 \\ -2 & n = -2 \\ -3 & n = -1 \\ -5 & n = 0 \\ -5 & n = 1 \\ 1 & n = 2 \\ 4 & n = 3 \\ 6 & n = 4 \\ 8 & n = 5 \\ 0 & n = 6 \end{cases}$$



Good Luck