ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester - Fall 2018
Solution
EL 313- Signal & Systems

Assignment – 4 **Due Date: 21/12/2018**
Marks: 10 **Handout Date: 14/12/2018**

Question # 1:

Compute the discrete-time Fourier transform of the following signals:

1. $x[n] = \left(\frac{1}{4}\right)^n u[n]$
2. $x[n] = \left(\frac{1}{3}\right)^n u[-n]$

Solution:

Q#1 :-

1) $x[n] = \left(\frac{1}{4}\right)^n u[n]$

Sol:-

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u[n] e^{-jwn} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n e^{-jwn}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4} e^{-jw}\right)^n \quad \because \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \text{ for } |a| < 1$$

$$X(e^{jw}) = \frac{1}{1 - \frac{1}{4} e^{-jw}}$$

$\longleftrightarrow$

2) $x[n] = \left(\frac{1}{3}\right)^n u[n]$

Sol:-

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n u[-n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n e^{-jwn} = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{-n} e^{jwn}$$

$$= \sum_{n=0}^{\infty} \left(3e^{jw}\right)^n \quad \because \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \text{ for } |a| < 1$$

$$X(e^{jw}) = \frac{1}{1 - 3e^{jw}}$$

Question # 2:

Find the Fourier series coefficients for each of the following signals:

1. $x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$
2. $x(t) = 1 + \cos(2\pi t)$

Solution:

Sol:

Using Euler's identity $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$$x(t) = \frac{e^{j(10\pi t + \pi/6)} - e^{-j(10\pi t + \pi/6)}}{2j}$$

$$= \frac{e^{j10\pi t} e^{j\pi/6}}{2j} - \frac{e^{-j10\pi t} e^{-j\pi/6}}{2j}$$

$$= \frac{e^{j\pi/6}}{2j} e^{j2\pi t 5} - \frac{e^{-j\pi/6}}{2j} e^{-j2\pi t 5}$$

$\therefore x(t) = \sum_k a_k e^{jk\omega_0 t}$

hence $\omega_0 = 2\pi$

then

$k = 5 \& -5$

$a_5 = \frac{e^{j\pi/6}}{2j}$, $a_{-5} = \frac{-e^{-j\pi/6}}{2j}$

otherwise $a_k = 0$.

2) $x(t) = 1 + \cos(2\pi t)$

Sol:

Using Euler's identity $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$$x(t) = 1 + \frac{e^{j2\pi t}}{2} + \frac{e^{-j2\pi t}}{2}$$

$\omega_0 = 2\pi$

$a_{-1} = a_1 = \frac{1}{2}$ and $a_0 = 1$. All other a_k's $= 0$.

Good Luck