ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester - Fall 2018
Solution
EL 313- Signal & Systems

Assignment – 5 **Due Date: 11/01/2019**
Marks: 20 **Handout Date: 31/12/2018**

Question # 1:

A particular LTI system is described by the difference equation:

$$y[n]+\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=x[n]-x[n-1]$$

Find the impulse response of the system.

Solution:

The use of the Fourier transform simplifies the analysis of the difference equation:

$$y[n]+\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=x[n]-x[n-1]$$

$$Y(e^{j\omega})+\frac{1}{4}e^{-j\omega}Y(e^{j\omega})-\frac{1}{8}e^{-2j\omega}Y(e^{j\omega})=X(e^{j\omega})-e^{-j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega})\left(1+\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)=X(e^{j\omega})\left(1-e^{-j\omega}\right)$$

$$H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1-e^{-j\omega}}{1+\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}}$$

$$H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1-e^{-j\omega}}{\left(1+\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

Using Partial fraction expansion, we see that:

$$H(e^{j\omega})=\frac{1-e^{-j\omega}}{\left(1+\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}=\frac{A}{\left(1+\frac{1}{2}e^{-j\omega}\right)}+\frac{B}{\left(1-\frac{1}{4}e^{-j\omega}\right)}\rightarrow eq1$$

Cross multiplication yields:

$$1-e^{-j\omega}=A\left(1-\frac{1}{4}e^{-j\omega}\right)+B\left(1+\frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega}=4$, gives:

$$1-4=A\left(1-\frac{1}{4}\times(4)\right)+B\left(1+\frac{1}{2}\times(4)\right)$$

$$1-4=A(0)+B(1+2)$$

$$-3=B(3)\Longrightarrow B=-1$$

$$1 = A\left(1 - \frac{3}{4}e^{-j\omega}\right) + B\left(1 - \frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = -2$, gives:

$$1 - (-2) = A\left(1 - \frac{1}{4}\times(-2)\right) + B\left(1 + \frac{1}{2}\times(-2)\right)$$

$$1 + 2 = A\left(\frac{2+1}{2}\right) + B(0)$$

$$3 = A\left(\frac{3}{2}\right) \Rightarrow A = 2$$

Putting values of A and B in eq(1) gives:

$$H(e^{j\omega}) = \frac{2}{\left(1 + \frac{1}{2}e^{-j\omega}\right)} + \frac{-1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

Taking the inverse Fourier transform, we obtain:

$$h[n] = 2\left(-\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n]$$

Question # 2:

Determine the z-transforms of the following two signals. Also sketch the pole-zero plot and ROC for each signal:

1. $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$
2. $x_2[n] = -\left(\frac{1}{2}\right)^n u[-n-1]$

Solution:

1. $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$

Solution:

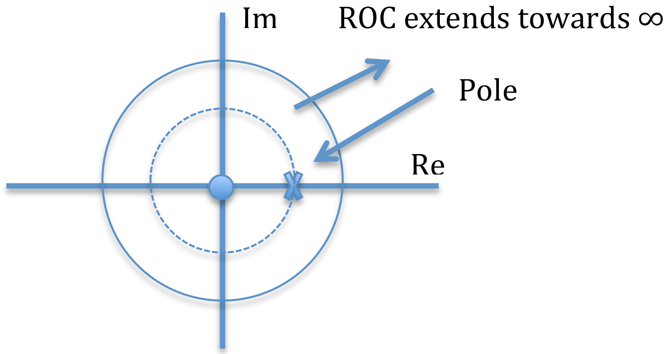
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n]z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} \Rightarrow \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$zeros\ is\ at\ 0\ and\ pole\ is\ at\ 1 - \frac{1}{2}z^{-1} = 0 \Rightarrow z = \frac{1}{2}$$

$$Region\ of\ Covergence =\ |z| > \frac{1}{2}$$



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2. $x_2[n] = -\left(\frac{1}{2}\right)^n u[-n-1]$

Solution:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} -\left(\frac{1}{2}\right)^n u[-n-]z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^n z^{-n}$$

Let n = -m, we have:

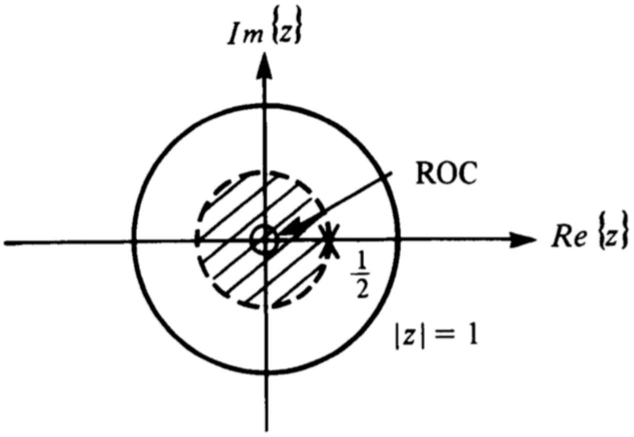
$$X(z) = -\sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^{-m} z^m$$

$$= -\sum_{m=1}^{\infty} (2z)^m$$

$$= -\frac{2z}{1-2z} \Longrightarrow \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$zeros\ is\ at\ 0\ and\ pole\ is\ at\ 1-\frac{1}{2}z^{-1} = 0 \Longrightarrow z = \frac{1}{2}$$

$$Region\ of\ Covergence = |2z| < 1, or\ |z| < \frac{1}{2}$$



Good Luck