day / date: THURS 3-01-19

LECTURE #10

:- SOLVED EXAMPLES :-

EXAMPLE #1:

$$x[n] = \delta[n] + \delta[n-1] + \delta[n+1]$$

SOLUTION:

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \left(\delta[n] + \delta[n-1] + \delta[n+1]\right) e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \delta[n] e^{-jwn} + \sum_{n=-\infty}^{\infty} \delta[n-1] e^{-jwn} + \sum_{n=-\infty}^{\infty} \delta[n+1] e^{-jwn}$$

$$X(e^{jw}) = 1 + e^{-jw} + e^{jw}$$

$$\Rightarrow 1 + \left[2\cos w\right]$$

EXAMPLE #2:

a) $x[n] = x[1-n] + x[-1-n]$

Sol:

Using the time reversal property, we have

$$x[-n] \xleftrightarrow{FT} X(e^{-jw})$$

Using the shifting property, we have

$$x[-n+1] \xleftrightarrow{FT} e^{-jwn} X(e^{-jw})$$

&

$$x[-1-n] \xleftrightarrow{FT} e^{jwn} X(e^{-jw})$$

Therefore,

$$x[n] = x[1-n] + x[-1-n] \xleftrightarrow{FT} e^{-jwn} X(e^{-jw}) + e^{jwn} X(e^{-jw})$$

$$\xleftrightarrow{FT} X(e^{-jw}) \left(e^{-jwn} + e^{jwn}\right)$$

$$\xleftrightarrow{FT} 2X(e^{-jw}) \cos w.$$

b) $x(n) = (n-1)^2 x(n)$

Sol:

Using the differentiation in frequency property, we have

$$n x(n) \xleftrightarrow{FT} j\frac{dX(e^{jw})}{dw}$$

Using the same property a second time,

$$n^2 x(n) \xleftrightarrow{FT} -\frac{d^2 X(e^{jw})}{dw^2}$$

Therefore,

$$x(n) = n^2 x(n) - 2n x(n) + 1 \xleftrightarrow{FT} -\frac{d^2 X(e^{jw})}{dw^2} - 2j\frac{dX(e^{jw})}{dw}$$

$$+ X(e^{jw}).$$

EXAMPLE # 3:-

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]$$

a) $H(e^{jw}) = ?$

Sol:-

$$Y(e^{jw}) - \frac{1}{6}Y(e^{jw})e^{-jw} - \frac{1}{6}e^{-2jw}Y(e^{jw}) = X(e^{jw})$$

$$Y(e^{jw})\left[1 - \frac{1}{6}e^{-jw} - \frac{1}{6}e^{-2jw}\right] = X(e^{jw})$$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} \Rightarrow \frac{1}{1 - \frac{1}{6}e^{-jw} - \frac{1}{6}e^{-2jw}}$$

$\therefore -\frac{1}{2} + \frac{1}{3} \Rightarrow -\frac{1}{6}$

$$H(e^{jw}) = \frac{1}{\left(1 - \frac{1}{2}e^{-jw}\right)\left(1 + \frac{1}{3}e^{-jw}\right)}$$

b) $h[n] = ?$

Sol:-

Using Partial Fraction

$$H(e^{jw}) = \frac{1}{\left(1 - \frac{1}{2}e^{-jw}\right)\left(1 + \frac{1}{3}e^{-jw}\right)} = \frac{A}{1 - \frac{1}{2}e^{-jw}} + \frac{B}{1 + \frac{1}{3}e^{-jw}}$$

Cross multiplication we get:

$$1 = A\left(1 + \frac{1}{3}e^{-jw}\right) + B\left(1 - \frac{1}{2}e^{-jw}\right)$$

let $e^{-j\omega} = -3$, $e^{-j\omega} = 2$

when $e^{-j\omega} = -3$

$$1 = A\left[1 + \cancel{\tfrac{1}{3}(-3)}\right] + B\left[1 - \tfrac{1}{2}(-3)\right]$$

$$1 = B\left[1 - \left(-\tfrac{3}{2}\right)\right]$$

$$1 = B\left[\frac{2+3}{2}\right]$$

$$B\left[\frac{5}{2}\right] = 1 \Rightarrow B = 2/5$$

when $e^{-j\omega} = 2$

$$1 = A\left[1 + \tfrac{1}{3}(2)\right] + B\left[1 - \cancel{\tfrac{1}{2}(2)}\right]$$

$$1 = A\left[\frac{3+2}{3}\right]$$

$$A = 3/5$$

$$H(e^{j\omega}) = \frac{3/5}{1 - \frac{1}{2}e^{-j\omega}} + \frac{2/5}{1 + \frac{1}{3}e^{-j\omega}}$$

Taking the inverse Fourier transform:-

$$h[n] = \frac{3}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(-\frac{1}{3}\right)^n u[n].$$