Lecture # 11
Solved Examples

Example #4:

$$X(z) = \frac{1}{1-az^{-1}}, \quad |z| > |a|$$

Expand in a power series by long division.

Sol:

$$X(z) = \frac{1}{1-az^{-1}}$$

By using long division

$$\begin{array}{r|l} & 1 + az^{-1} + a^2z^{-2} + \cdots \\ \hline 1-az^{-1} & 1 \\ & \underline{-1 + az^{-1}} \\ & az^{-1} \\ & \underline{-az^{-1} + a^2z^{-2}} \\ & a^2z^{-2} \\ & \underline{-a^2z^{-2} + a^3z^{-3}} \\ & a^3z^{3} \\ & \vdots \end{array}$$

or

$$\frac{1}{1-az^{-1}} = 1 + az^{-1} + a^2z^{-2} + \cdots$$

→ The series expansion converges since $|z| > |a|$ or equivalently $|az^{-1}| < 1$

⇒ By matching terms in power of z, we see that $x[n] = 0$ $n < 0$, $x[0] = 1$ $x[1] = a$, $x[2] = a^2$ and in general $x(n) = a^n u(n)$