ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester - Fall 2018
Solution
EE313- Signal & Systems

Quiz – 3
Marks: 30 **Handout Date: 11/01/2019**

Question # 1:

Consider an LTI system for which the input x [n] and the output y[n] satisfy the linear constant coefficient difference equation:

$$y[n]-\frac{1}{2}y[n-1]=x[n]+\frac{1}{3}x[n-1]$$

Solution:

The use of the Fourier transform simplifies the analysis of the difference equation:

$$y[n]-\frac{1}{2}y[n-1]=x[n]+\frac{1}{3}x[n-1]$$

$$Y(e^{j\omega})-\frac{1}{2}e^{-j\omega}Y(e^{j\omega})=X(e^{j\omega})+\frac{1}{3}e^{-j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega})\left(1-\frac{1}{2}e^{-j\omega}\right)=X(e^{j\omega})\left(1+\frac{1}{3}e^{-j\omega}\right)$$

$$H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1+\frac{1}{3}e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}$$

$$H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1}{1-\frac{1}{2}e^{-j\omega}}+\frac{1}{3}\frac{e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}$$

Taking the inverse Fourier transform of $H(e^{j\omega})$, we get:

$$h[n]=\left(\frac{1}{2}\right)^{n}u[n]+\frac{1}{3}\left(\frac{1}{2}\right)^{n-1}u[n-1]$$

Question # 2:

Determine the Fourier series coefficients of the following signal:

$$x[n] = \cos\tfrac{\pi}{4}n$$

Solution:

$$x[n] = \cos\frac{\pi}{4}n$$

Using Euler's identity:

$$x[n] = \frac{e^{\frac{j\pi}{4}n}}{2} + \frac{e^{-\frac{j\pi}{4}n}}{2}$$

The fundamental frequency, $\omega_0 = \frac{\pi}{4}$.

Where:

$$a_1 = \frac{1}{2}\,, a_{-1} = \frac{1}{2}$$

Otherwise $a_k = 0$.

Hence, the discrete Fourier series of x [n] is:

$$x[n] = \frac{e^{\frac{j\pi}{4}n}}{2} + \frac{e^{-\frac{j\pi}{4}n}}{2}$$

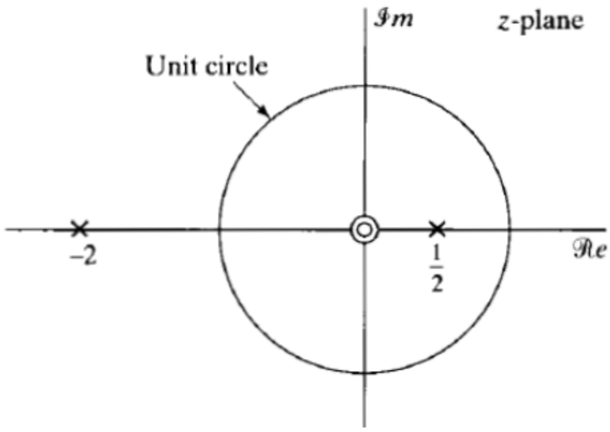
Question # 3:

a) Consider a system with system function:

$$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}\ , |z| > 2$$

Find h [n] and also tell whether the system is Causal or not?

b) For the following pole-zero plot, determine that whether the system is both causal and stable or not. If not, then explain why?



Solution:

a) Consider a system with system function:

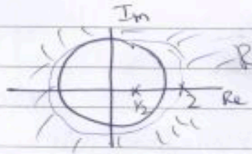
$$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}\ , |z| > 2$$

Find h [n] and also tell whether the system is Causal or not?

Solution:

$$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}\ , |z|>2$$

$h[n] = ?$



Sol:-

⇒ Since the ROC for this system function is the exterior of a circle outside the outermost pole, we know that the impulse response is right sided.

⇒ To determine if the system is causal, we then need only check the other condition required for causality, namely that H(z), when expressed as a ratio of polynomials in z, has numerator degree no larger than the denominator.

→ For this example:

$$H(z) = \frac{1-2z^{-1} + 1 - \frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})} = \frac{2 - \frac{5}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

$$H(z) \Rightarrow \frac{2z^2 - 5/2\, z}{z^2 - \frac{5}{2}z - 1}$$

→ The numerator and denominator of $H(z)$ are both of degree two, and consequently we can conclude that the system is causal.

→ this can also be verified by calculating the inverse transform of $H(z)$.

→ Using pair 5 in table of standard z-transform pairs, the impu response of this system is,

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + (2)^n u[n].$$

→ Since $h[n] = 0$ for $n < 0$, we can confirm that the system is causal.

Stability:-

→ If we consider the example #5 for checking stability of the system we see that since the ROC associated is the region $|z| > 2$ which does not include the unit circle, the system is not stable.

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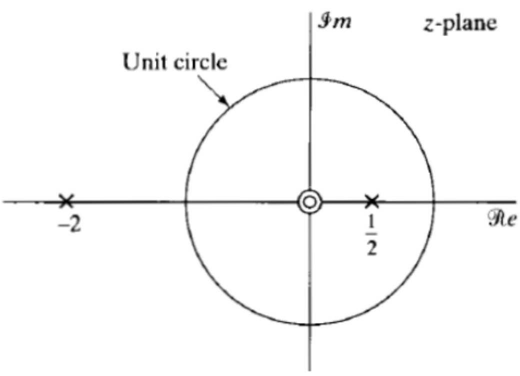
b) For the following pole-zero plot, determine that whether the system is both causal and stable or not. If not, then explain why?



Solution:

A system is causal, as the ROC will extend outwards the outermost pole to infinity.
A system is stable, as the unity circle is included in the ROC.

Good Luck