Name: **Regd. No.**

Course Title: Signal & Systems **Course Code:** EE-314

MID SEMESTER EXAMINATION - FALL 2018

Program: BSC & MSC (Electrical)

Solution

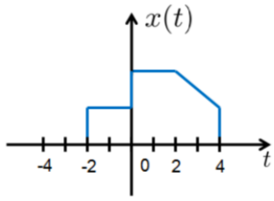
SECTION-II: 20 MARKS

Time Allowed: 1hr 15 min

Attempt all questions. Marks are mentioned against the questions. **Note: Please attach the question paper at the end of the answer sheet.**

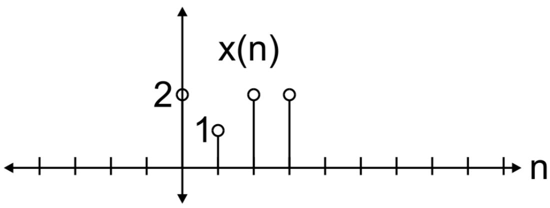
Q1.

a) A continuous time signal is shown below. Sketch and label the signal $x(2t-2)$:



(2.5 Marks)

b) A discrete time signal is shown below. Sketch and label the signal $-x[n-3]$:

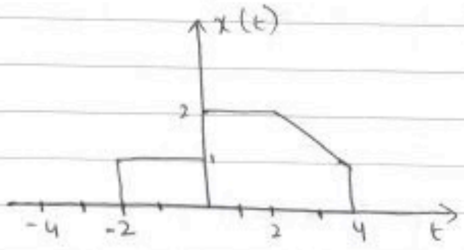


(2.5 Marks)

Solution:

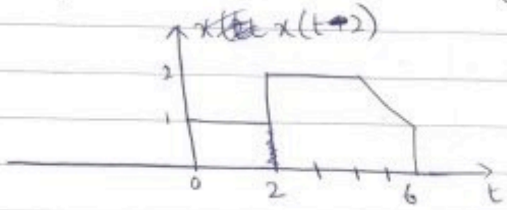
Q#1:-

a) $x(2t-2) = ?$

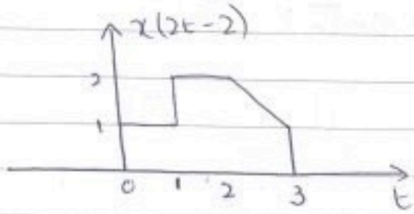


Sol:-

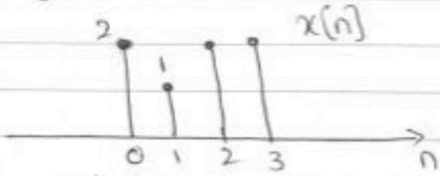
First shift $x(t)$ 2 points towards right



Now scale it by 2.

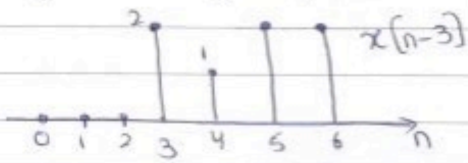


b) $-x[n-3]$

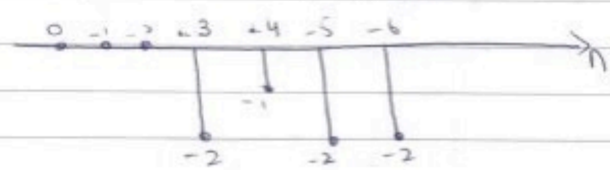


Sol:-

First shift $x[n]$ by 3 points towards right



Now flip ~~y(n)~~ $x[n-3]$ along y-axis



Q2. Determine whether or not the following signals are periodic. If a signal is periodic, determine its fundamental period.

$$x(t) = \cos t + \sin \sqrt{2}t$$

(05 Marks)

Solution:

$x_1(t) = \cos t$, $x_2(t) = \sin\sqrt{2}\,t$

$T_1 = \frac{2\pi}{\omega_1}$ $\because \omega_1 = 1$, $T_2 = \frac{2\pi}{\omega_2}$ $\therefore \omega_2 = \sqrt{2}$

$T_1 = 2\pi$, $T_2 = \frac{2\pi}{\sqrt{2}}$

Now check the ratio of T_1/T_2 :-

$\frac{T_1}{T_2} = \frac{2\pi}{2\pi/\sqrt{2}} \Rightarrow \frac{2\pi}{2\pi} \times \sqrt{2} \Rightarrow \sqrt{2}$

Hence, $\frac{T_1}{T_2}$ is an irrational number, $x(t)$ is non-periodic.

Q3. Classify the following signals into energy, power or neither. Determine energy and power.

i. $x(t) = e^{2t}u(-t)$
ii. $x[n] = (-0.5)^n u[n]$

(05 Marks)

Solution:

1) $x(t) = e^{2t} u(-t)$

Sol^n

$$E = \int_{-\infty}^{\infty} (e^{2t})^2 u(-t)\, dt$$

$$= \int_{-\infty}^{0} e^{4t}\, dt$$

$$E = \frac{e^{4t}}{4}\Big|_{-\infty}^{0} = \frac{e^{(0)4}}{4} - \cancel{\frac{e^{(-\infty)4}}{4}}^{\,0}$$

$$E = \frac{e^{0}}{4} \Rightarrow \frac{1}{4} < \infty$$

Hence, it is an energy signal.

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2) $x[n] = (-0.5)^n u[n]$

Sol^n

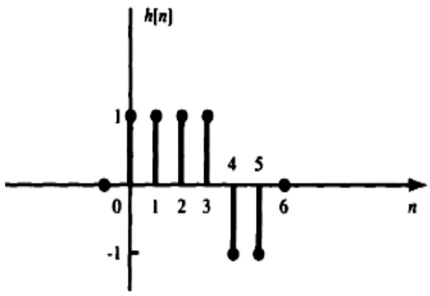
$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

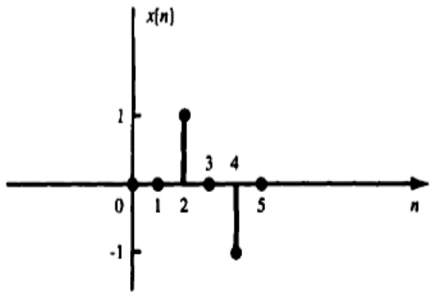
$$= \sum_{n=0}^{\infty} (-0.5)^{n2}$$

$$= \sum_{n=0}^{\infty} (0.25)^n = \frac{1}{1-0.25} \Rightarrow \frac{4}{3} < \infty$$

Hence, it is an energy signal.

Q4. Evaluate $y[n] = x[n] * h[n]$.

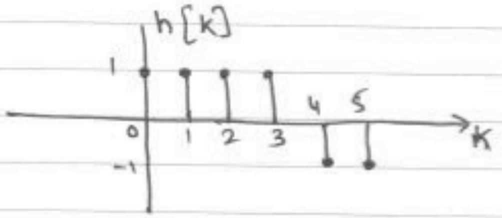


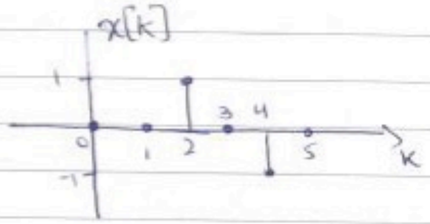


(05 Marks)

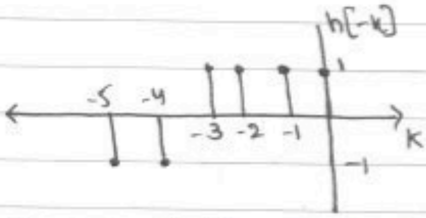
Solution:

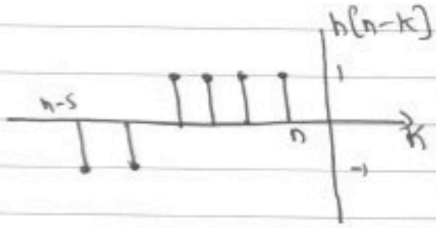
Step #1 $n \rightarrow k$



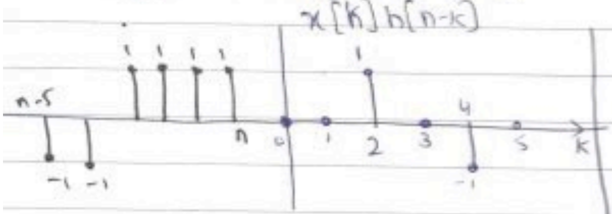


Step #2 Flip and shift $h[k]$





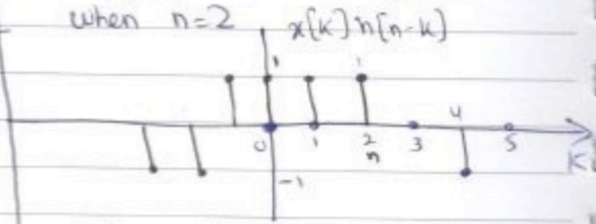
Step #3 Starting overlapping



when $n<0$ $y[n]=0$, as there is no overlapping

overlapping will start from $n=2$.

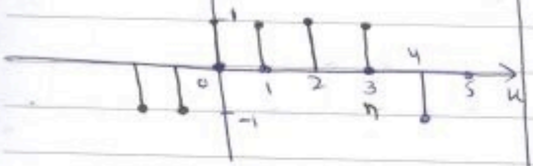
when $n=2$



$y[2] = (1 \times 1) \Rightarrow 1.$

when $n=3$

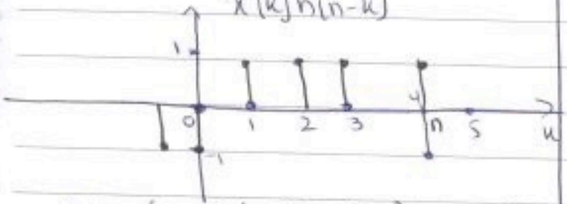
$x[k]\,h[n-k]$



$$y[3] = (1\times1) + (0\times1) \Rightarrow 1$$

when $n=4$

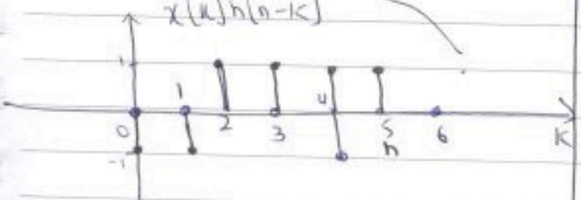
$x[k]\,h[n-k]$



$$y[4] = (0\times-1)+(0\times1)+(1\times1)+(0\times1)+(1\times-1)$$
$$= 0+0+\cancel{1}+0-\cancel{1} \Rightarrow 0$$

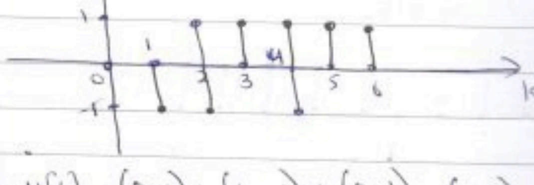
when $n=5$

$x[k]\,h[n-k]$



$$y[5] = (0\times-1)+(0\times-1)+(1\times1)+(0\times1)+(-1\times1)+(0\times1)$$
$$= 0+0+1+0-1+0 = 0$$

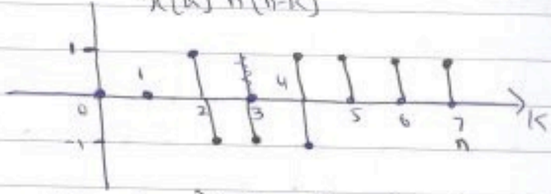
when $n=6$ $x[k]\,h[n-k]$



$$y[6] = (0\times1)+(1\times-1)+(0\times1)+(-1\times1)+(0\times1)+(0\times1)$$
$$= 0-1+0-1+0+0 \Rightarrow -2$$

when $n=7$

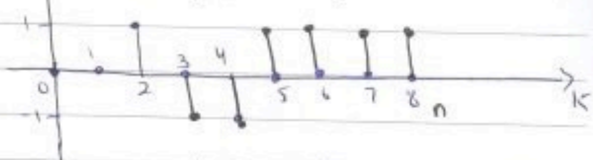
$x[k]\,h[n-k]$



$$y[7] = (+1\times-1)+(0\times-1)+(-1\times1)+(0\times1)+(0\times1)+(0\times1) = -1+0-1+0+0+0 \Rightarrow -2$$

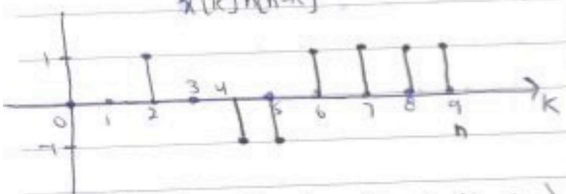
when $n=8$

$x[k]\,h[n-k]$



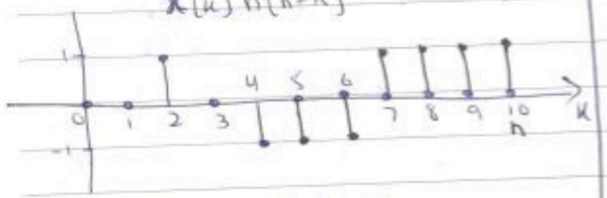
$$y[8] = (0\times-1)+(-1\times-1)+(0\times1)\,(0\times1)+(0\times1)+(0\times1)$$
$$= 0+1+0+0+0+0 \Rightarrow 1$$

When $n=9$



$y[9] = (-1\times -1) + (0\times -1) + (0\times 1) + (0\times 1)$
$+ (0\times 1) + (0\times 1)$
$= +1 + 0 + 0 + 0 + 0 + 0 \Rightarrow 1$

When $n=10$

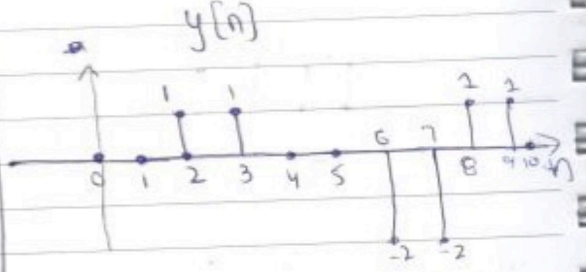


There is no overlapping.
$y[10] = 0$

Step #4 Draw $y(n)$.

Now,

$$y[n] = \begin{cases} n<0 & 0 \\ n=0 & 0 \\ n=1 & 0 \\ n=2 & 1 \\ n=3 & 1 \\ n=4 & 0 \\ n=5 & 0 \\ n=6 & -2 \\ n=7 & -2 \\ n=8 & 1 \\ n=9 & 1 \\ n=10 & 0 \end{cases}$$



Formula Sheet

S. No.	Continuous-Time	Discrete-Time
1.	$Frequency : \; f = \frac{1}{T}$ $Angular\ Frequency : \; \omega = 2\pi f = \frac{2\pi}{T}$ $Fundamental\ Period : \; T = \frac{2\pi}{\omega}$	$Angular\ Frequency : \; \omega = \frac{2\pi k}{N}$ $Fundamental\ Period : \frac{N}{k} = \frac{2\pi}{\omega}$
2.	$Energy : \; E = \int_{-\infty}^{\infty} [x(t)]^2 dt$	$Energy : \; E = \sum_{n=-\infty}^{\infty} \lvert x[n] \rvert^2$
	$Power : \; P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [x(t)]^2 dt$ If x (t) is periodic, then its average power becomes: $P = \frac{1}{T}\int_0^T [x(t)]^2 dt$	$Power : \; P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x[n] \rvert^2$
3.	Convolution Integral $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$	Convolution Sum $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$