Name: **Regd. No.**

Course Title: Signal & Systems **Course Code:** EE-313

MID SEMESTER EXAMINATION – Fall 2018

Program: B.E. (Electrical)

Solution

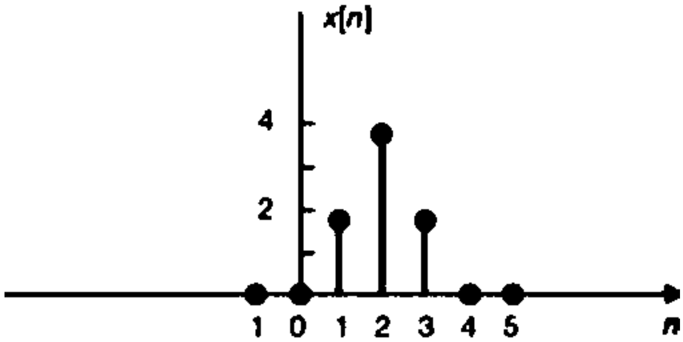
SECTION-II: 24 MARKS

Time Allowed: 1hr 10 min

Attempt all questions. Marks are mentioned against the questions. **Note: Please attach the question paper at the end of the answer sheet.**

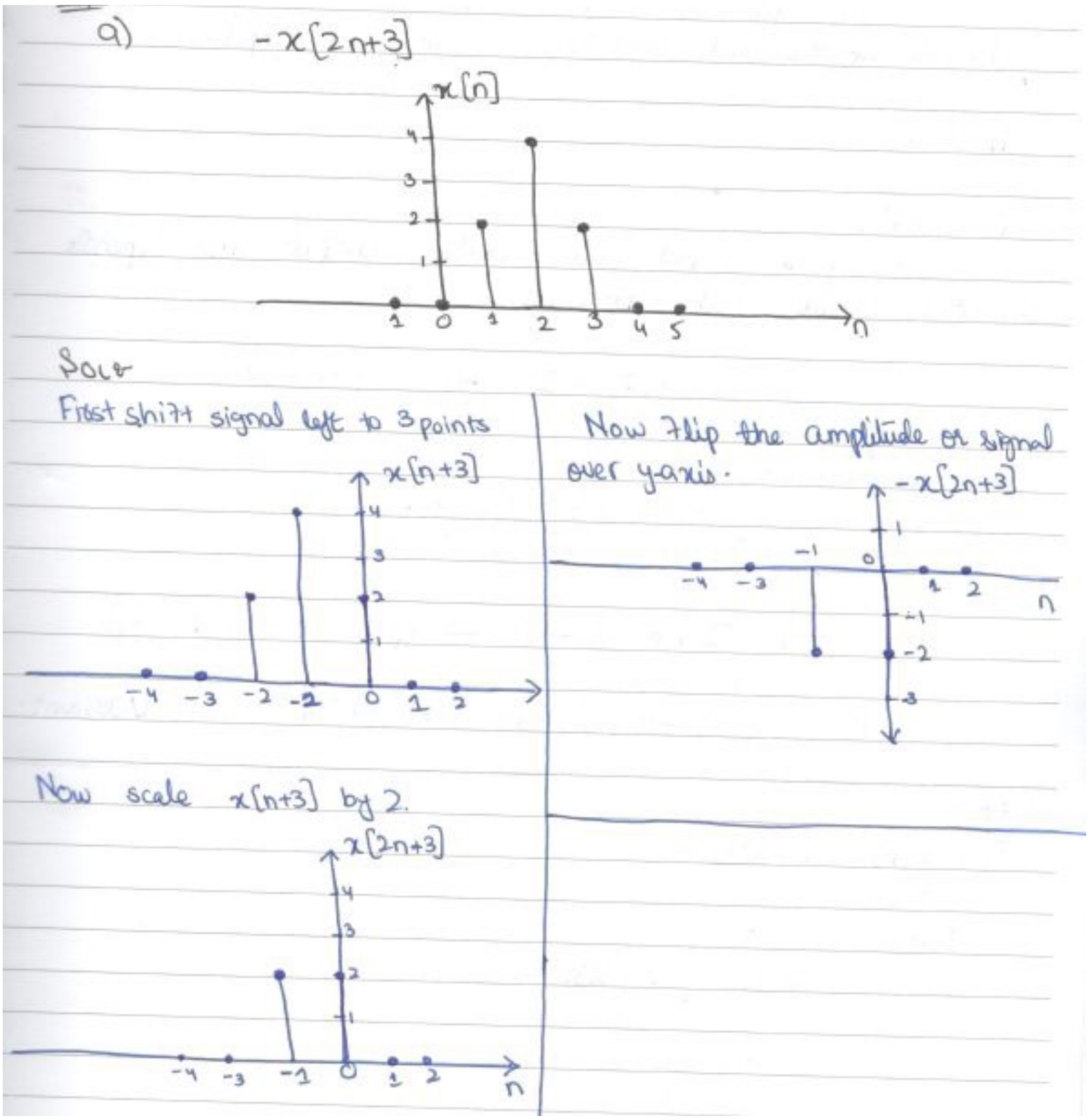
Q1.

a) A discrete time signal is shown below. Sketch and label the signal $-x[2n+3]$:



(03 Marks)

Solution:



b) Determine whether the following system $y[n] = nx[2n]$ is:

- **i.** Memory-less
- **ii.** Causal
- **iii.** Time-invariant

(03 Marks)

Solution:

b) $y[n] = nx[2n]$

i. Memoryless?

The system is not memoryless as the output ~~depen~~ depends on the past and future values of the input.

~~iii. Stable~~

ii. Causal?

The system is not causal as the output also depends on future values of the input.

$y[2] = 2x[4]$ Non-Causal

iii. Timeinvariant?

$x[n] \rightarrow \boxed{SYS} \rightarrow y[n] = nx[2n]$.

$y[n] \xrightarrow{n-n_0} (n-n_0)x[2(n-n_0)] \Rightarrow y'[n]$

$x[n] \xrightarrow{n_0} x[n-n_0] \rightarrow \boxed{SYS} \rightarrow y_1[n] = nx[2(n-n_0)]$

As $y'[n] \neq y_1[n]$, hence the system is time variant.

Q2. Determine whether or not the following signals are periodic. If a signal is periodic, determine its fundamental period.

i. $x[n] = \sin\left(\frac{6\pi n}{7} + 1\right)$

ii. $x[n] = e^{j\frac{3\pi}{5}\left(n+\frac{1}{5}\right)}$

(06 Marks)

Solution:

i) $x[n] = \sin\left(\frac{6\pi n}{7} + 1\right)$

Sol:-

$x[n] = \sin\left(\frac{6\pi n}{7} + 1\right)$

$\omega_0 = \frac{6\pi}{7}$

$\frac{N}{m} = \frac{2\pi}{\omega_0}$

$$\frac{N}{m} = \frac{2\pi}{6\pi/7} \Rightarrow \frac{2\pi}{3\cdot 6\pi} \times 7$$

$$\frac{N}{m} = \frac{7}{3}$$

$N=7$, is the fundamental period of $x[n]$.

ii- $x[n] = e^{j\frac{3\pi}{5}(n+\frac{1}{5})}$

Sol:-

$$x[n] = e^{j\left(\frac{3\pi n}{5} + \frac{3\pi}{25}\right)}$$

$$\omega_0 = \frac{3\pi}{5}$$

$$\frac{N}{m} = \frac{2\pi}{\omega_0}$$

$$\frac{N}{m} = \frac{2\pi}{3\pi/5} \Rightarrow \frac{2\pi}{3\pi} \times 5$$

$$\frac{N}{m} = \frac{10}{3}$$

$N=10$, is the fundamental period of $x[n]$.

Q3. Classify the following signals into energy, power or neither. Determine energy and power.

i. $x(t) = e^{2t}u(-t)$
ii. $x(t) = t\,u(t)$

(06 Marks)

Solution:

i) $x(t) = e^{2t}\, u(-t)$

Sol:-

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-\infty}^{\infty} (e^{2t})^2 u(-t)\, dt = \int_{-\infty}^{0} e^{4t}\, dt$$

$$E = \frac{e^{4t}}{4}\Big|_{-\infty}^{0} = \frac{e^{-4(0)}}{4} - \frac{e^{-\infty}}{4} \Rightarrow +\frac{1}{4} < \infty$$

Hence it is an energy signal.

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ii) $x(t) = e^{-3|t|}$

Sol:

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left(e^{-3|t|}\right)^2 dt$$

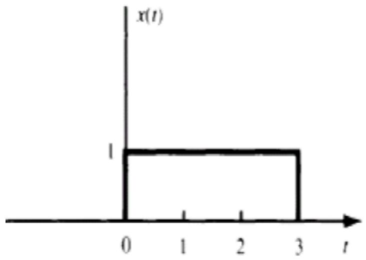
$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} e^{-6|t|}\, dt$$

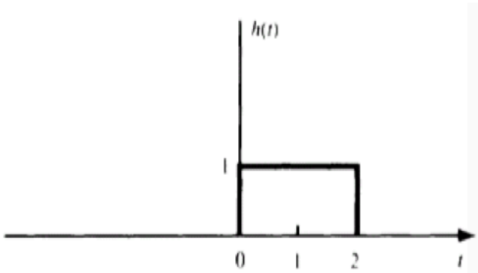
$$= \lim_{T \to \infty} \frac{1}{2T} \left[-\frac{e^{-6|t|}}{6} \right]_{-T}^{T}$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \left[-\frac{e^{-6T}}{6} + \frac{e^{+6T}}{6} \right] \Rightarrow \infty$$

After putting the $\lim_{T \to \infty}$ the signal is neither power nor energy signal.

Q4. Evaluate $y(t) = x(t) * h(t)$, where $x(t) = u(t) - u(t-3)$ and $h(t) = u(t) - u(t-2)$ by using graphical method.



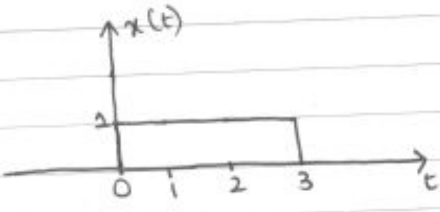


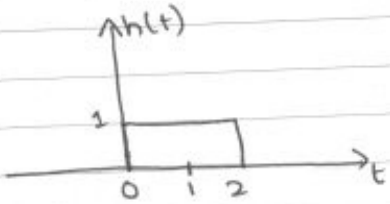
(06 Marks)

Solution:

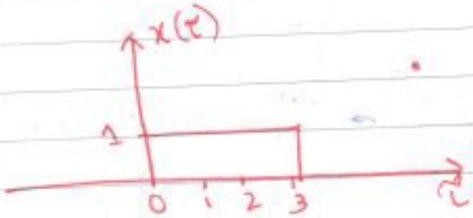
Q# 1:

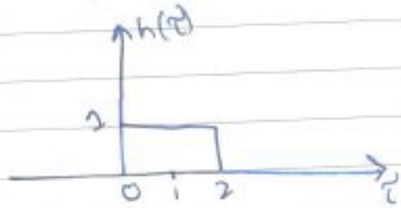
$x(t) = u(t) - u(t-3)$ and $h(t) = u(t) - u(t-2)$.

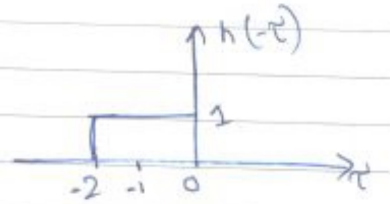




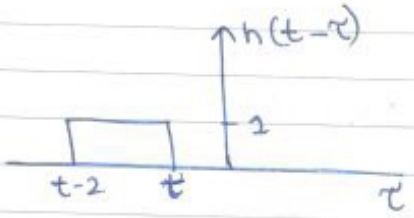
Sol:-

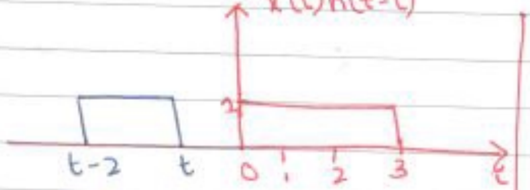






$\Longleftrightarrow$

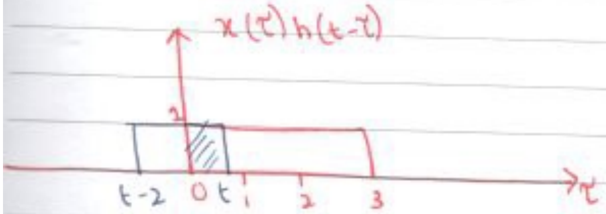




when $t < 0$

$y(t) = 0$ as no overlapping

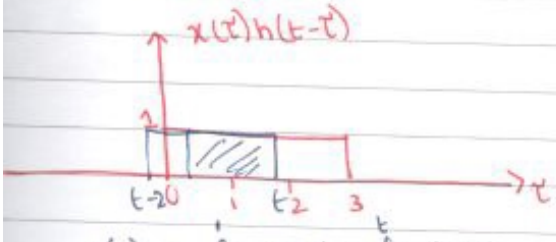
when $0 < t \leq 1$



$$y(t) = \int_0^t (1 \times 1)\, d\tau = \tau\Big|_0^t$$

$$y(t) = t - 0 \Rightarrow t$$

when $1 < t < 2$



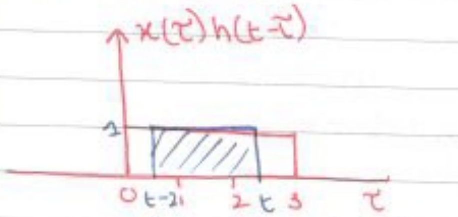
$$y(t) = \int_0^1 (1\times 1) d\tau + \int_1^t (1 \times 1) d\tau$$

$$= \int_0^1 d\tau + \int_1^t d\tau = \tau\Big|_0^1 + \tau\Big|_1^t$$

$$y(t) = (1-0) + (t-1)$$

$$= \cancel{1} + t \cancel{-1} \Rightarrow t$$

When $2 < t \leq 3$

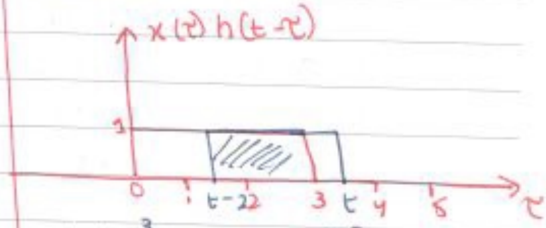


$$y(t) = \int_{t-2}^t 1\, d\tau = [\tau]_{t-2}^t$$

$$= t - (t-2) \Rightarrow \cancel{t} - \cancel{t} + 2$$

$$y(t) = 2$$

when $3 < t \leq 4$



$$y(t) = \int_{t-2}^3 1\, d\tau = [\tau]_{t-2}^3$$

$$y(t) = 3 - t + 2 \Rightarrow 5 - t$$

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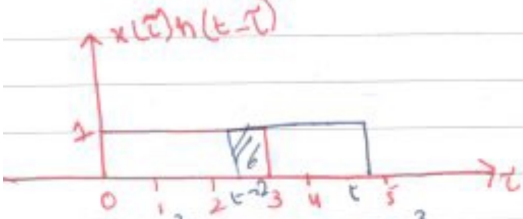
when $4 < t \le 5$

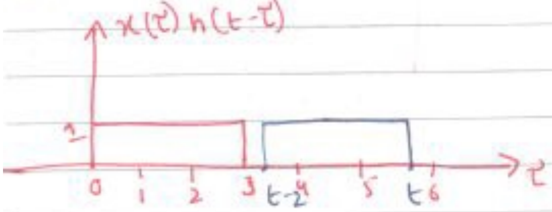


$$y(t) = \int_{t-2}^{3} 1 \, d\tau = [\tau]_{t-2}^{3}$$

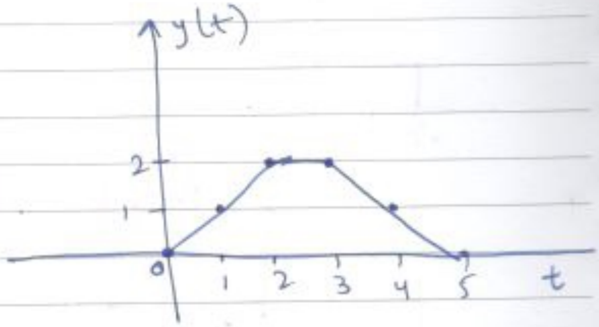
$$= 3 - t + 2 \Rightarrow 5 - t$$

when $t > 5$

$y(t) = 0$ hence there will be no overlapping



$$y(t) = \begin{cases} 0 & , t < 0 \\ t & , 0 < t \le 1 \\ t & , 1 < t \le 2 \\ 2 & , 2 < t \le 3 \\ 5-t & , 3 < t \le 4 \\ 5-t & , 4 < t \le 5 \\ 0 & , t > 5 \end{cases}$$



Formula Sheet

S. No.	Continuous-Time	Discrete-Time
1.	$Frequency: \ f = \frac{1}{T}$ $Angular\ Frequency: \ \omega = 2\pi f = \frac{2\pi}{T}$ $Fundamental\ Period: \ T = \frac{2\pi}{\omega}$	$Angular\ Frequency: \ \omega = \frac{2\pi k}{N}$ $Fundamental\ Period: \frac{N}{k} = \frac{2\pi}{\omega}$
2.	$Energy: \ E = \int_{-\infty}^{\infty} [x(t)]^2 dt$	$Energy: \ E = \sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$
	$Power: \ P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [x(t)]^2 dt$ If x (t) is periodic, then its average power becomes: $P = \frac{1}{T}\int_0^T [x(t)]^2 dt$	$Power: \ P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x[n]\rvert^2$
3.	Convolution Integral $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$	Convolution Sum $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$