Name: **Regd. No.**

Course Title: Signal & Systems **Course Code:** EL-313

FINAL TERM EXAMINATION – Fall 2016
Program: B.E. (Electrical)
"Solution"
SECTION-II: 40 MARKS
Time Allowed: 2hr 10 min

Attempt all questions. Marks are mentioned against the questions. **Note: Please attach the question paper at the end of the answer sheet.**

Q1. Find the Fourier series coefficients for each of the following signals:

i. $x(t) = sin\left(10\pi t + \frac{\pi}{6}\right)$

ii. $x(t) = 1 + \frac{1}{2}cos(2\pi t) + \sin(3\pi t)$

(05 Marks)

Solution:

i. $x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$
Using Euler's identity:

$$x(t) = \frac{e^{j\pi/6}}{2j}e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j}e^{-j2\pi t5}$$

The fundamental frequency, $\omega_0 = 2\pi$.

$$x(t) = \sum_k a_k e^{jk\omega_0 t}$$

Where:

$$a_5 = \frac{e^{j\pi/6}}{2j} \;, a_{-5} = \frac{-e^{-j\pi/6}}{2j}$$

Otherwise $a_k = 0$.

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ii. $x(t) = 1 + \frac{1}{2}cos(2\pi t) + \sin(3\pi t)$

Fundamental Period:

$$T_1 \Rightarrow 1, \qquad T_2 = \frac{2\pi}{2\pi} \Rightarrow 1, \qquad T_3 = \frac{2\pi}{3\pi} \Rightarrow \frac{2}{3}$$

$$\frac{T_1}{T_2} \Rightarrow 1, \qquad \frac{T_1}{T_3} = {}^{1}/_{2/3} \Rightarrow \frac{3}{2}$$

$$T = LCM\left(1, \frac{3}{2}\right) \Rightarrow 2$$

Using Euler's identity:

$$x(t) = 1 + \frac{1}{4}\left[e^{j2\pi t} + e^{-j2\pi t}\right] + \frac{1}{2j}\left[e^{j3\pi t} - e^{-j3\pi t}\right]$$

$$x(t) = 1 + \frac{1}{4}e^{j2\pi t} + \frac{1}{4}e^{-j2\pi t} + \frac{1}{2j}e^{j3\pi t} - \frac{1}{2j}e^{-j3\pi t}$$

The fundamental frequency, $\omega_0 = \pi$.

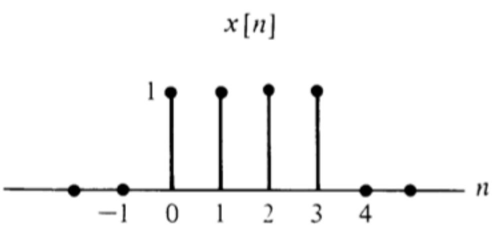
$$x(t) = \sum_k a_k e^{jk\omega_0 t}$$

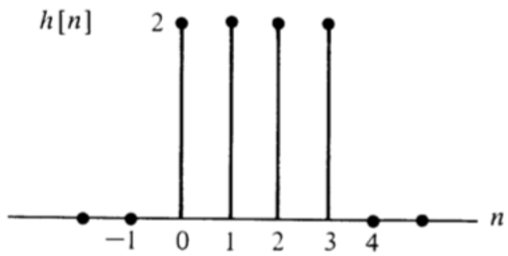
Where:

$$a_0 = 1, \qquad a_1 = a_{-1} = 0, \qquad a_2 = a_{-2} = \frac{1}{4}, \qquad a_3 = \frac{1}{2j}, \qquad a_{-3} = -\frac{1}{2j}$$

Otherwise $a_k = 0, |k| > 3$

Q2. Determine the discrete-time convolution of x [n] and h [n] for the following case:

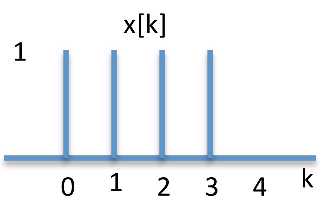


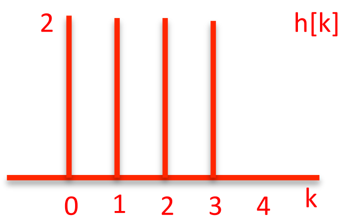


(05 Marks)

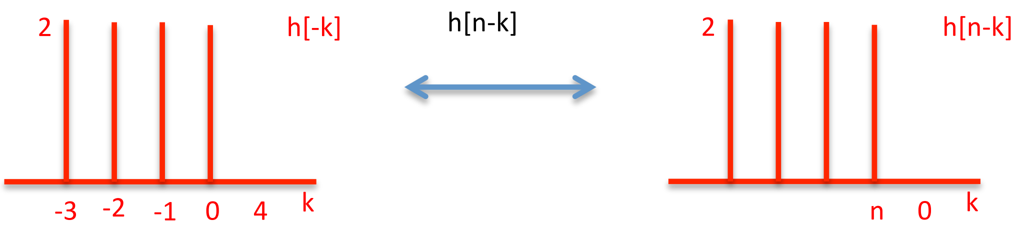
Solution:

Step#1: Change the subscript n to k.

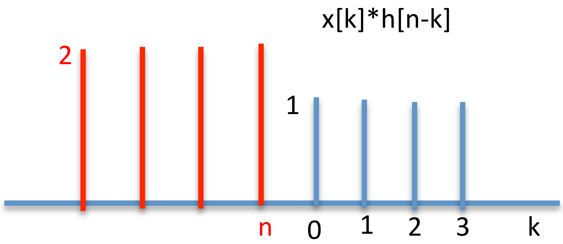




Step#2: Flip and shift anyone of the signal. Here we are flipping and shifting h[k].

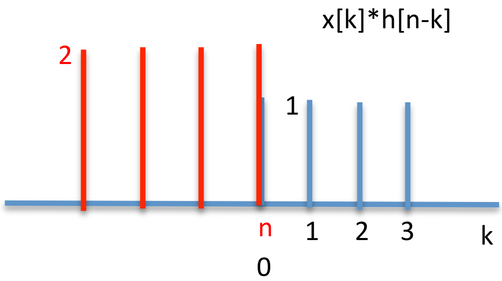


Step#3: Start sliding h [n-k] over the signal x [k] and convolve.



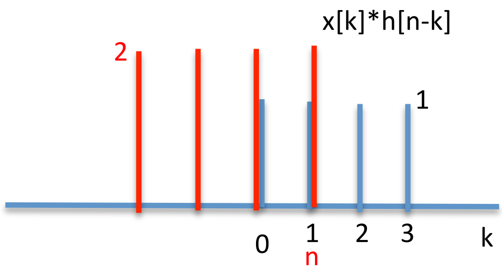
$$when\; n < 0$$

$$\sum_{n=-\infty}^{\infty} x[k]h[n-k] = 0 \; As\; there\; is\; no\; overlapping$$



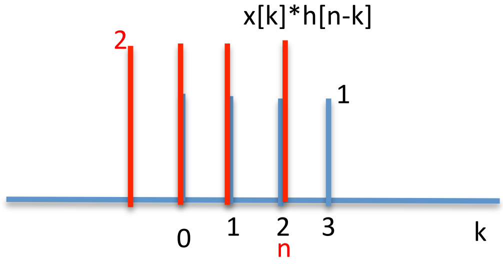
$$when\; n = 0$$

$$y[0] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = 2\times(1) \Longrightarrow 2$$



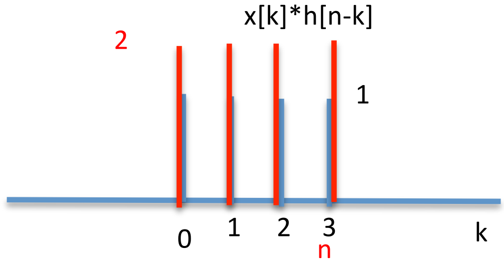
$$when\; n = 1$$

$$y[1] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times1) + (2\times1) \Longrightarrow 4$$



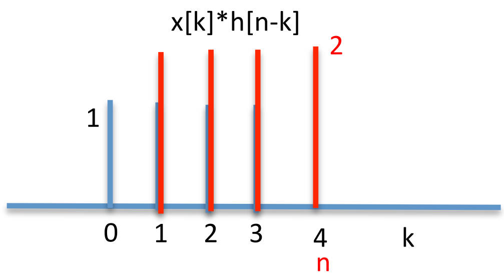
$$when\ n = 2$$

$$y[2] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times1) + (2\times1) + (2\times1) = 2 + 2 + 2 \Longrightarrow 6$$



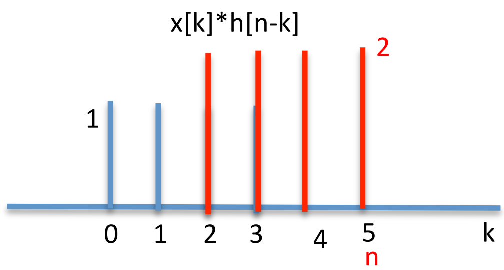
$$when\ n = 3$$

$$y[3] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times1) + (2\times1) + (2\times1) + (2\times1) = 2 + 2 + 2 + 2 \Longrightarrow 8$$



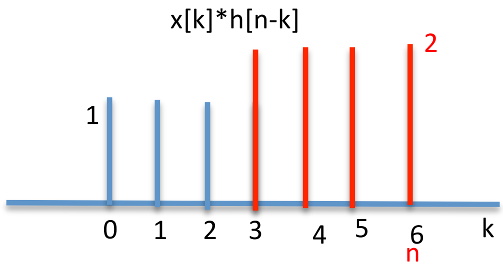
$$when\ n = 4$$

$$y[4] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times0) + (2\times1) + (2\times1) + (2\times1) = 0 + 2 + 2 + 2 \Longrightarrow 6$$



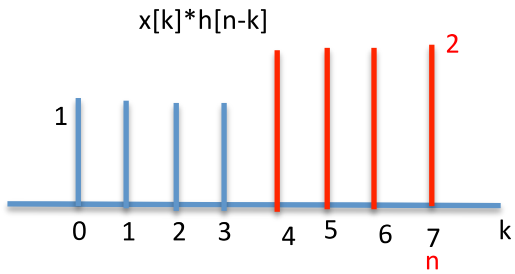
$$when\ n = 5$$

$$y[5] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times0) + (2\times0) + (2\times1) + (2\times1) = 0 + 0 + 2 + 2 \Longrightarrow 4$$



$$when\ n = 6$$

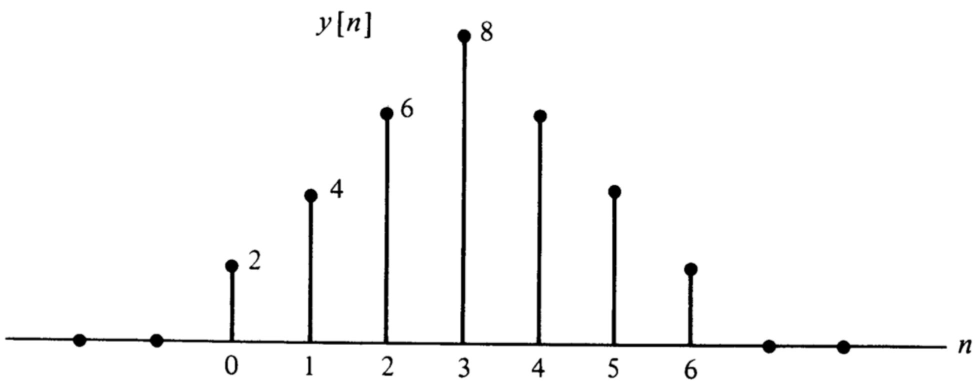
$$y[6] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times0) + (2\times0) + (2\times0) + (2\times1) = 0 + 0 + 0 + 2 \Longrightarrow 2$$



$$when\ n = 7$$

$$y[7] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = 0\ As\ there\ is\ no\ overlapping$$

Step#4: Sketch the final signal.



Q3. Consider the signal $x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t - 10\cos 12000\pi t$:

- **i.** What is the Nyquist rate for this signal?
- **ii.** Using a sampling rate $F_s = 5000\ \text{samples/s}$. What is the Discrete-time signal obtained after sampling?

(05 Marks)

Solution:

i. What is the Nyquist rate for this signal?

$$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t - 10\cos 12000\pi t$$

$$F_1 = \frac{2\text{K}\pi}{2\pi} \Rightarrow 1\text{KHz}, \qquad F_2 = \frac{6\text{K}\pi}{2\pi} \Rightarrow 3\text{KHz}, \qquad F_3 = \frac{12\text{K}\pi}{2\pi} \Rightarrow 6\text{KHz}$$

The Nyquist rate is $F_N = 2F_{max} = 2\times 6KHz \Rightarrow 12KHz$

ii. Using a sampling rate $F_s = 5000\ \text{samples/s}$. What is the Discrete-time signal obtained after sampling?

$$F_s = 5\text{KHz}$$

$$x[n] = x_a(nT) = x_a(n/F_s)$$

$$= 3\cos 2\pi\left(\frac{1}{5}\right)n + 5\sin 2\pi\left(\frac{3}{5}\right)n - 10\cos 2\pi\left(\frac{6}{5}\right)n$$

$$= 3\cos 2\pi\left(\frac{1}{5}\right)n + 5\sin 2\pi\left(1-\frac{2}{5}\right)n - 10\cos 2\pi\left(1+\frac{1}{5}\right)n$$

$$x[n] = -7\cos 2\pi\left(\frac{1}{5}\right)n - 5\sin 2\pi\left(\frac{2}{5}\right)n$$

$$F_s = 5\text{KHz}$$

$$F_{max} = \frac{5\text{K}}{2} \Rightarrow 2.5KHz$$

Hence,
$F_1 = 1\text{KHz}$ Is not effected by aliasing.
$F_2 = 3\text{KHz}$ Is changed by the aliasing effect $F_2' = F_2 - F_s = 3\text{K} - 5\text{K} \Rightarrow -2\text{KHz}$
$F_3 = 6\text{KHz}$ Is changed by the aliasing effect $F_3' = F_3 - F_s = 6\text{K} - 5\text{K} \Rightarrow 1\text{KHz}$

So, that the normalize frequencies are: $f_1 = \frac{1}{5},\ f_2 = \frac{2}{5},\ f_3 = \frac{1}{5}$

Q4. Determine the Fourier Transform of following signals using Analysis equation:

- **i.** $x[n] = \left(\frac{1}{4}\right)^n u[n]$
- **ii.** $x[n] = \begin{cases} -1, & n = -3,-1,1,3 \\ 1, & n = -2,0,2 \\ 0, & otherwise \end{cases}$

(05 Marks)

Solution:

i. $x[n] = \left(\frac{1}{4}\right)^n u[n]$

Solution:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\left(\frac{1}{4}\right)^n u[n]\right)e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{4} e^{-j\omega}\right)^n$$

$$Using\ the\ fact: \sum_{n=0}^{\infty} (a)^n = \frac{1}{1-a}, for\ |a| < 1$$

$$X(e^{j\omega}) = \frac{1}{1 - \frac{1}{4} e^{-j\omega}}$$

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ii. $x[n] = \begin{cases} -1, & n = -3, -1, 1, 3 \\ 1, & n = -2, 0, 2 \\ 0, & otherwise \end{cases}$

Solution:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-3}^{3} x[n] e^{-j\omega n}$$

$$= x[-3]e^{j3\omega} + x[-2]e^{j2\omega} + x[-1]e^{j\omega} + x[0]e^{0} + x[1]e^{-j\omega} + x[2]e^{-j2\omega} + x[3]e^{-j3\omega}$$

$$= (-1)e^{j3\omega} + (1)e^{j2\omega} + (-1)e^{j\omega} + (1)1 + (-1)e^{-j\omega} + (1)e^{-j2\omega} + (-1)e^{-j3\omega}$$

$$= -e^{j3\omega} + e^{j2\omega} - e^{j\omega} + 1 - e^{-j\omega} + e^{-j2\omega} - e^{-j3\omega}$$

$$= 1 - \left(e^{j3\omega} + e^{-j3\omega}\right) + \left(e^{j2\omega} + e^{-j2\omega}\right) - \left(e^{j\omega} + e^{-j\omega}\right)$$

Using Euler's identity: $\cos x = \frac{e^{jx} + e^{-jx}}{2}$ $or\ 2\cos x = e^{jx} + e^{-jx}$

$$X(e^{j\omega}) \Rightarrow 1 - 2\cos(3\omega) + 2\cos(2\omega) - 2\cos(\omega)$$

Q5. A particular LTI system is described by the difference equation:

$$y[n] + \frac{1}{4} y[n-1] - \frac{1}{8} y[n-2] = x[n] - x[n-1]$$

Find the impulse response h [n] of the system.

(05 Marks)

Solution:

The use of the Fourier transform simplifies the analysis of the difference equation:

$$y[n] + \frac{1}{4} y[n-1] - \frac{1}{8} y[n-2] = x[n] - x[n-1]$$

$$Y(e^{j\omega}) + \frac{1}{4} e^{-j\omega} Y(e^{j\omega}) - \frac{1}{8} e^{-2j\omega} Y(e^{j\omega}) = X(e^{j\omega}) - e^{-j\omega} X(e^{j\omega})$$

$$Y(e^{j\omega}) \left(1 + \frac{1}{4} e^{-j\omega} - \frac{1}{8} e^{-2j\omega}\right) = X(e^{j\omega}) \left(1 - e^{-j\omega}\right)$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - e^{-j\omega}}{1 + \frac{1}{4}e^{-j\omega} - \frac{1}{8}e^{-2j\omega}}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - e^{-j\omega}}{\left(1 + \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

Using Partial fraction expansion, we see that:

$$H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{\left(1 + \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{1}{4}e^{-j\omega}\right)} = \frac{A}{\left(1 + \frac{1}{2}e^{-j\omega}\right)} + \frac{B}{\left(1 - \frac{1}{4}e^{-j\omega}\right)} \rightarrow eq1$$

Cross multiplication yields:

$$1 - e^{-j\omega} = A\left(1 - \frac{1}{4}e^{-j\omega}\right) + B\left(1 + \frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = 4$, gives:

$$1 - 4 = A\left(1 - \frac{1}{4}\times(4)\right) + B\left(1 + \frac{1}{2}\times(4)\right)$$

$$1 - 4 = A(0) + B(1 + 2)$$

$$-3 = B(3) \Rightarrow B = -1$$

$$1 = A\left(1 - \frac{3}{4}e^{-j\omega}\right) + B\left(1 - \frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = -2$, gives:

$$1 - (-2) = A\left(1 - \frac{1}{4}\times(-2)\right) + B\left(1 + \frac{1}{2}\times(-2)\right)$$

$$1 + 2 = A\left(\frac{2+1}{2}\right) + B(0)$$

$$3 = A\left(\frac{3}{2}\right) \Rightarrow A = 2$$

Putting values of A and B in eq(1) gives:

$$H(e^{j\omega}) = \frac{2}{\left(1 + \frac{1}{2}e^{-j\omega}\right)} + \frac{-1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

Taking the inverse Fourier transform, we obtain:

$$h[n] = 2\left(-\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n]$$

Q6. Determine the z-transform for each of the following sequences. Sketch the pole-zero plot and indicate the ROC:

i. $\left(\frac{1}{2}\right)^n u[n]$

ii. $(-1)^n u[n]$

(05 Marks)

Solution:

i. $\left(\frac{1}{2}\right)^n u[n]$

Solution:

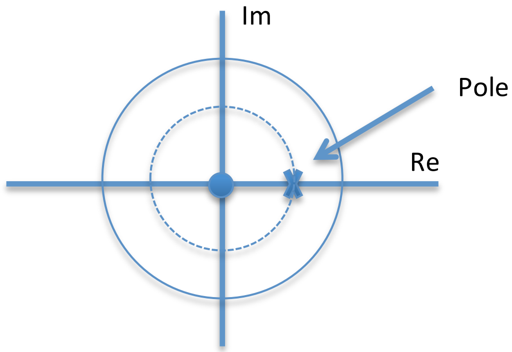
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n]z^{-1}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-1} \Rightarrow \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$zeros\ is\ at\ 0\ and\ pole\ is\ at\ 1-\frac{1}{2}z^{-1} = 0 \Rightarrow z = \frac{1}{2}$$

$$Region\ of\ Covergence = \ |z| > \frac{1}{2}$$



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ii. $(-1)^n u[n]$

Solution:

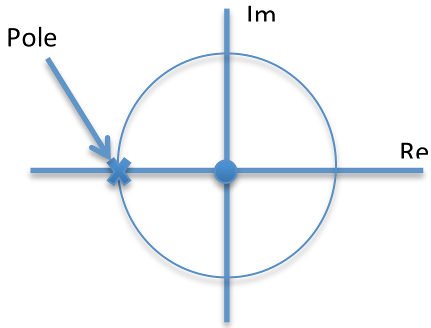
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (-1)^n u[n]z^{-1}$$

$$= \sum_{n=0}^{\infty} (-1)^n z^{-1} \Rightarrow \frac{1}{1+z^{-1}}$$

$$zeros\ is\ at\ 0\ and\ pole\ is\ at\ 1+z^{-1} = 0 \Rightarrow z = -1$$

$$Region\ of\ Covergence = \ |z| > 1$$



Q7. Suppose we are given X (z) as following:

$$X(z) = \frac{3 - \frac{5}{6}z^{-1}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)}$$

Find the inverse z-Transform i.e., x [n]=? Also, sketch the pole-zero plot.

(05 Marks)

Solution:

There are two poles, one at z=1/3 and one at z=1/4. The ROC lies outside the outermost pole.

Using partial fraction expansion:

$$X(z) = \frac{3 - \frac{5}{6}z^{-1}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)} = \frac{A}{\left(1 - \frac{1}{4}z^{-1}\right)} + \frac{B}{\left(1 - \frac{1}{3}z^{-1}\right)} \rightarrow eq(1)$$

Cross multiplication gives:

$$3 - \frac{5}{6}z^{-1} = A\left(1 - \frac{1}{3}z^{-1}\right) + B\left(1 - \frac{1}{4}z^{-1}\right)$$

Putting $z^{-1} = 3$, gives:

$$3 - \frac{5}{6}(3) = A\left(1 - \frac{1}{3}(3)\right) + B\left(1 - \frac{1}{4}(3)\right)$$

$$3 - \frac{5}{2} = A(1 - 1) + B\left(1 - \frac{3}{4}\right)$$

$$\frac{6 - 5}{2} = 0 + B\left(\frac{4 - 3}{4}\right)$$

$$\frac{1}{2} = B\left(\frac{1}{4}\right) \Longrightarrow B = \frac{1}{2} \times 4 = 2$$

Putting $z^{-1} = 4$, gives:

$$3-\frac{5}{6}(4) = A\left(1-\frac{1}{3}(4)\right) + B\left(1-\frac{1}{4}(4)\right)$$

$$3-\frac{10}{3} = A\left(1-\frac{4}{3}\right) + B(1-1)$$

$$\frac{9-10}{3} = A\left(\frac{3-4}{3}\right) + 0$$

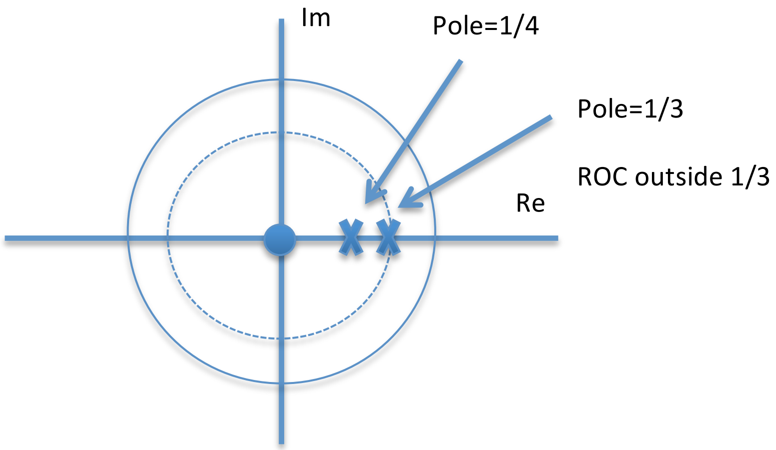
$$-\frac{1}{3} = A\left(-\frac{1}{3}\right) \Rightarrow A = -\frac{1}{3}\times(-3) = 1$$

Putting values of A and B in eq(1) gives:

$$X(z) = \frac{1}{\left(1-\frac{1}{4}z^{-1}\right)} + \frac{2}{\left(1-\frac{1}{3}z^{-1}\right)}$$

Taking the inverse z-transform, we obtain:

$$x[n] = \left(\frac{1}{4}\right)^n u[n] + 2\left(\frac{1}{3}\right)^n u[n]$$

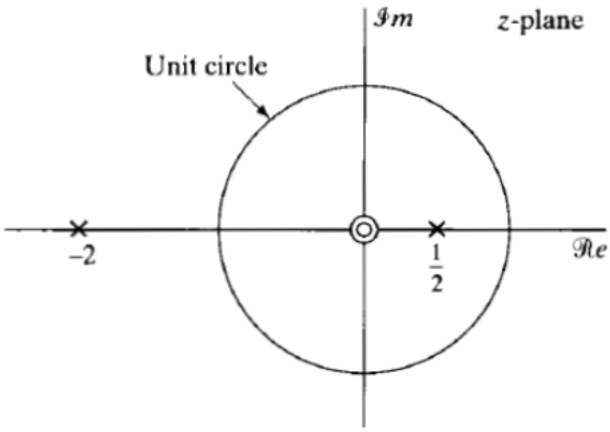


Q8.

a) Draw the direct form (block diagram) of the system H (z) mentioned below:

$$H(z) = \frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

b) For the following pole-zero plot, determine that whether the system is both causal and stable or not. If not, then explain why?



(03+02 Marks)

Solution:

a) Draw the direct form (block diagram) of the system H (z) mentioned below:

$$H(z) = \frac{1}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

Solution:

$$H(z) = \frac{1}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)} = \frac{1}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

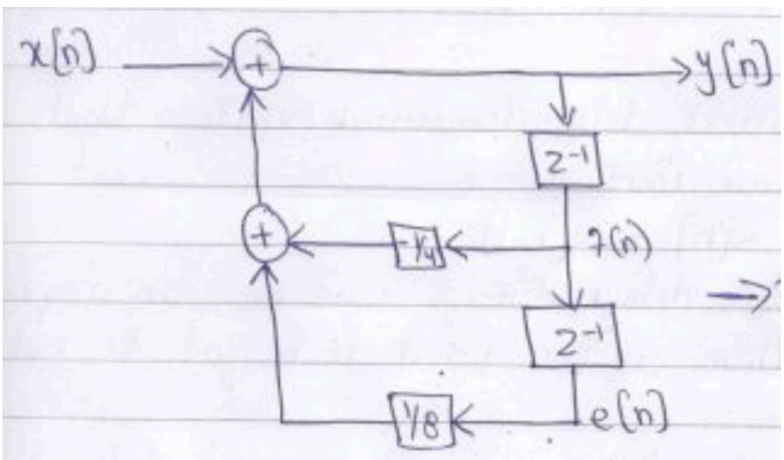
The associated difference equation of H (z) is:

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n]$$

Which can be rewritten as:

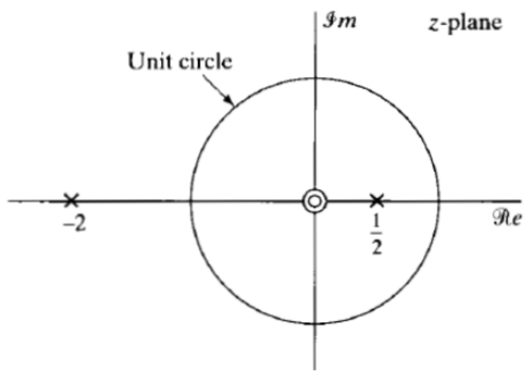
$$y[n] = -\frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + x[n]$$

Block diagram representation for the system H (z) is:



Where: $f[n] = y[n-1]$ & $e[n] = f[n-1] = y[n-2]$

b) For the following pole-zero plot, determine that whether the system is both causal and stable or not. If not, then explain why?



Solution:

A system is causal, as the ROC will extend outwards the outermost pole to infinity.
A system is stable, as the unity circle is included in the ROC.

Formula Sheet

S. No.	Continuous-Time	Discrete-Time
1.	$Frequency: \ f = \frac{1}{T}$ $Angular\ Frequency: \ \omega = 2\pi f = \frac{2\pi}{T}\ or\ f = \frac{\omega}{2\pi}$ $Fundamental\ Period: \ T = \frac{2\pi}{\omega}$	$Angular\ Frequency: \ \omega = \frac{2\pi k}{N}$ $Fundamental\ Period: \frac{N}{k} = \frac{2\pi}{\omega}$
2.	Convolution Integral $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$	Convolution Sum $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$
	Fourier Series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\ , Synthesis\ Equation$	Fourier Series $x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n}\ , Synthesis\ Equation$
3.	$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt\ , Analysis\ Equation$	$a_k = \frac{1}{N}\sum_{n=\langle N \rangle} x[n]e^{-jk\omega_0 n}\ , Analysis\ Equation$

$$Nyquist\ Rate: \ F_N = 2F_{max}$$

Properties of the Discrete Time Fourier Transform

Non-periodic signal	*Fourier transform*
$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$	$X(e^{j\omega}) \triangleq \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$	$\left.\begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix}\right\}$ Periodic with period 2π
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
$x^*[n]$ $x[-n]$	$X^*(e^{j(-\omega)})$ $X(e^{j(-\omega)})$
$x_{(m)}[n] = \begin{cases} x[n/m], & n \text{ multiple of } m \\ 0, & n \text{ not multiple av } m \end{cases}$	$X(e^{j(m\omega)})$
$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
$x[n] - x[n-1]$	$\left(1 - e^{j\omega}\right)X(e^{j\omega})$
$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-e^{j}\omega}X(e^{j\omega}) + \pi X(0)\sum_{k=-\infty}^{\infty}\delta(\omega - 2\pi k)$
$nx[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$
If $x[n]$ is real valued then	
$x[n]$	$\begin{cases} X(e^{j\omega}) = X^*(e^{j(-\omega)}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{j(-\omega)})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{j(-\omega)})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{j(-\omega)})\rvert \\ \arg\{X(e^{j\omega})\} = -\arg\{X(e^{j(-\omega)})\} \end{cases}$
$x_e[n] = \mathcal{E}\{x[n]\}$ $x_o[n] = \mathcal{O}\{x[n]\}$	$\Re\{X(e^{j\omega})\}$ $j\Im\{X(e^{j\omega})\}$

Parsevals relation for non-periodic signals

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

Basic Pairs of Discrete Time Fourier Transform

$x[n]$	$X(e^{j\omega})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$\sum_{k=-\infty}^{\infty} \delta(n-kN)$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$
1	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$e^{j\omega_o n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_o - 2\pi k)$
$\cos \omega_o n$	$\pi \sum_{k=-\infty}^{\infty} [\delta(\omega-\omega_o-2\pi k) + \delta(\omega+\omega_o-2\pi k)]$
$\sin \omega_o n$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} [\delta(\omega-\omega_o-2\pi k) - \delta(\omega+\omega_o-2\pi k)]$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$a^n u(n), \quad \|a\| < 1$	$\frac{1}{1-ae^{-j\omega}}$
$(n+1)a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{(n+m-1)!}{n!(m-1)!} a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1-ae^{-j\omega})^m}$
$\frac{1}{1-a^2} a^{\|n\|}, \quad \|a\| < 1$	$\frac{1}{1+a^2-2acos\omega}$
$\begin{cases} 1, & \|n\| \leq N_1 \\ 0, & N_1 < \|n\| \leq \frac{N}{2} \end{cases}$ period N	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega \frac{2\pi k}{N}\right)$
$\begin{cases} 1, & \|n\| \leq N_1 \\ 0, & \|n\| > N_1 \end{cases}$	$\frac{\sin \omega \left(N_1 + \frac{1}{2}\right)}{\sin \frac{\omega}{2}}$
$\begin{cases} \frac{\sin Wn}{\pi n} = \frac{W}{\pi} \text{sinc} \frac{Wn}{\pi} \\ 0 < W < \pi \end{cases}$	$\begin{cases} 1, & \|\omega\| \leq W \\ 0, & W < \|\omega\| \leq \pi \end{cases}$ period 2π

Z-Transform Pairs

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3. $-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6. $-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7. $na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8. $-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9. $[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10. $[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11. $[r^n\cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
12. $[r^n\sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{1-a^Nz^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

Z-Transform Properties

signal	*Z-transform*	*ROC*
$x[n]$	$X(z) \triangleq \sum_{n=-\infty}^{\infty} x[n]z^{-n}$	R_x
$ax[n] + by[n]$	$aX(z) + bY(z)$	Contains $R_x \cap R_y$
$x[n - n_0]$	$z^{-n_0}X(z)$	R_x, except possible addition or deletion of the origin or ∞
$z_0^n x[n]$	$X(z/z_0)$	$\lvert z_0 \rvert R_x$
$x^*[n]$	$X^*(z^*)$	R_x
$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
$x[n] * y[n]$	$X(z)Y(z)$	Contains $R_x \cap R_y$
$nx[n]$	$-z\frac{d}{dz}X(z)$	R_x, except possible addition or deletion of the origin or ∞
$\Re\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	Contains R_x
$\Im\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	Contains R_x

Initial value theorem

$x[n] = 0, \; n < 0 \quad \lim_{z \to \infty} X(z) = x[0]$