Name: ____________________ **Regd. No.** ____________________

Course Title: Signal & Systems **Course Code:** ETSS-314

FINAL TERM EXAMINATION – Spring 2018
Program: B-Tech (Electrical)

SECTION-II: 40 MARKS
Time Allowed: 2hr 10 min

Attempt all questions. Marks are mentioned against the questions. **Note: Please attach the question paper at the end of the answer sheet.**

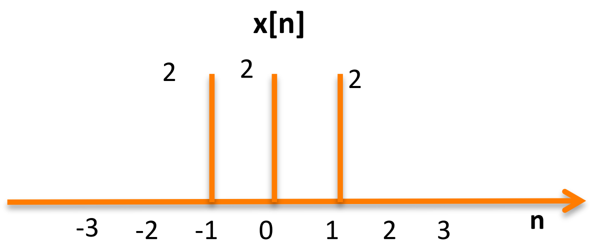
Q2.

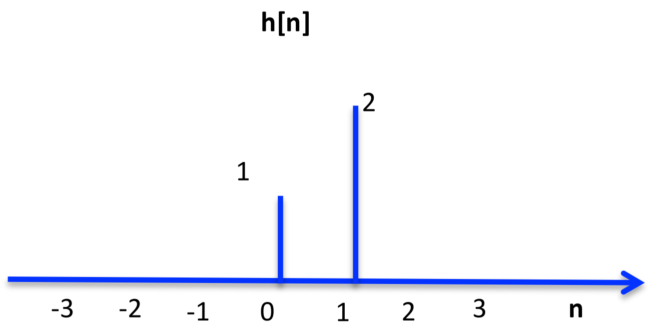
a) Find the Fourier Series Coefficients of the following signal:

$$x[n] = sin(\omega_0 n), where\ \omega_0 = \frac{2\pi}{N}$$

(05 Marks)

b) Evaluate $y[n] = x[n] * h[n]$. Using Graphical Method.





(08 Marks)

Q3.

a) Consider a discrete-time LTI system with impulse response:

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Use the Fourier transforms to determine the response y [n] to the given input:

$$x[n] = \left(\frac{3}{4}\right)^n u[n]$$

(08 Marks)

b) Consider the linear constant coefficient difference equation:

$$y[n] - \frac{1}{4}y[n-1] = x[n]$$

Which describes a linear, time-invariant system initially at rest. Determine $H(e^{j\omega})$.

(06 Marks)

Q4.

a) Determine the z-transform (including the ROC) of the following sequences. Also sketch the pole-zero plots and indicate the ROC on your sketch:

$$\left(\frac{1}{4}\right)^n u[n]$$

(05 Marks)

b) Using the partial fraction expansion, determine the sequence x [n] that goes with the following z-transform:

$$X(z) = \frac{3z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

(08 Marks)

Formula Sheet

S. No.	Continuous-Time	Discrete-Time
1.	$Frequency: \ f = \frac{1}{T}$	$Angular\ Frequency: \ \omega = \frac{2\pi k}{N}$ $Fundamental\ Period: \frac{N}{k} = \frac{2\pi}{\omega}$
	$Angular\ Frequency: \ \omega = 2\pi f = \frac{2\pi}{T} \ or\ f = \frac{\omega}{2\pi}$	
	$Fundamental\ Period: \ T = \frac{2\pi}{\omega}$	
2.	Convolution Integral $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$	Convolution Sum $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$
	Fourier Series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \ , Synthesis\ Equation$	Fourier Series $x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \ , Synthesis\ Equation$
3.	$a_k = \frac{1}{T}\int_{T} x(t)e^{-jk\omega_0 t}dt \ , Analysis\ Equation$	$a_k = \frac{1}{N}\sum_{n=\langle N \rangle} x[n]e^{-jk\omega_0 n} \ , Analysis\ Equation$

$$Nyquist\ Rate: \ F_N = 2F_{max}$$

Properties of the Discrete Time Fourier Transform

Non-periodic signal	*Fourier transform*
$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$	$X(e^{j\omega}) \triangleq \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$	$\left.\begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix}\right\}$ Periodic with period 2π
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
$x^*[n]$ $x[-n]$	$X^*(e^{j(-\omega)})$ $X(e^{j(-\omega)})$
$x_{(m)}[n] = \begin{cases} x[n/m], & n \text{ multiple of } m \\ 0, & n \text{ not multiple av } m \end{cases}$	$X(e^{j(m\omega)})$
$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
$x[n] - x[n-1]$	$\left(1 - e^{j\omega}\right) X(e^{j\omega})$
$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-e^{j}\omega}X(e^{j\omega}) + \pi X(0)\sum_{k=-\infty}^{\infty}\delta(\omega - 2\pi k)$
$nx[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$

If $x[n]$ is real valued then

$x[n]$	$\begin{cases} X(e^{j\omega}) = X^*(e^{j(-\omega)}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{j(-\omega)})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{j(-\omega)})\} \\ \left\lvert X(e^{j\omega})\right\rvert = \left\lvert X(e^{j(-\omega)})\right\rvert \\ \arg\{X(e^{j\omega})\} = -\arg\{X(e^{j(-\omega)})\} \end{cases}$
$x_e[n] = \mathcal{E}\{x[n]\}$ $x_o[n] = \mathcal{O}\{x[n]\}$	$\Re\{X(e^{j\omega})\}$ $j\Im\{X(e^{j\omega})\}$

Parsevals relation for non-periodic signals

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

Basic Pairs of Discrete Time Fourier Transform

$x[n]$	$X(e^{j\omega})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$\sum_{k=-\infty}^{\infty} \delta(n-kN)$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$
1	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$e^{j\omega_o n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_o - 2\pi k)$
$\cos \omega_o n$	$\pi \sum_{k=-\infty}^{\infty} [\delta(\omega - \omega_o - 2\pi k) + \delta(\omega + \omega_o - 2\pi k)]$
$\sin \omega_o n$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} [\delta(\omega - \omega_o - 2\pi k) - \delta(\omega + \omega_o - 2\pi k)]$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$a^n u(n), \quad \lvert a \rvert < 1$	$\frac{1}{1-ae^{-j\omega}}$
$(n+1)a^n u[n], \quad \lvert a \rvert < 1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{(n+m-1)!}{n!(m-1)!} a^n u[n], \quad \lvert a \rvert < 1$	$\frac{1}{(1-ae^{-j\omega})^m}$
$\frac{1}{1-a^2} a^{\lvert n \rvert}, \quad \lvert a \rvert < 1$	$\frac{1}{1+a^2-2acos\omega}$
$\begin{cases} 1, & \lvert n \rvert \le N_1 \\ 0, & N_1 < \lvert n \rvert \le \frac{N}{2} \end{cases}$ period N	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega \frac{2\pi k}{N}\right)$
$\begin{cases} 1, & \lvert n \rvert \le N_1 \\ 0, & \lvert n \rvert > N_1 \end{cases}$	$\frac{\sin \omega \left(N_1 + \frac{1}{2}\right)}{\sin \frac{\omega}{2}}$
$\begin{cases} \frac{\sin Wn}{\pi n} = \frac{W}{\pi} \text{sinc} \frac{Wn}{\pi} \\ 0 < W < \pi \end{cases}$	$\begin{cases} 1, & \lvert \omega \rvert \le W \\ 0, & W < \lvert \omega \rvert \le \pi \end{cases}$ period 2π

Z-Transform Pairs

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\dfrac{1}{1-z^{-1}}$	$\|z\| > 1$
3. $-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\|z\| < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
6. $-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
7. $na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
8. $-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
9. $[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
10. $[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
11. $[r^n \cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\|z\| > r$
12. $[r^n \sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\|z\| > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{1-a^N z^{-N}}{1-az^{-1}}$	$\|z\| > 0$

Z-Transform Properties

signal	*Z-transform*	*ROC*
$x[n]$	$X(z) \triangleq \sum_{n=-\infty}^{\infty} x[n]z^{-n}$	R_x
$ax[n] + by[n]$	$aX(z) + bY(z)$	Contains $R_x \cap R_y$
$x[n - n_0]$	$z^{-n_0}X(z)$	R_x, except possible addition or deletion of the origin or ∞
$z_0^n x[n]$	$X(z/z_0)$	$\lvert z_0 \rvert R_x$
$x^*[n]$	$X^*(z^*)$	R_x
$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
$x[n] * y[n]$	$X(z)Y(z)$	Contains $R_x \cap R_y$
$nx[n]$	$-z\frac{d}{dz}X(z)$	R_x, except possible addition or deletion of the origin or ∞
$\Re\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	Contains R_x
$\Im\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	Contains R_x

Initial value theorem

$x[n] = 0, \; n < 0 \quad \lim_{z\to\infty} X(z) = x[0]$