day / date: TUE/26th Feb, 19

LECTURE #2

:- SOLVED EXAMPLES:-

EXAMPLE #1:-

$$y = \frac{c}{x} \qquad \text{ODE } xy' = -y.$$

Sol:-

Differentiate $y = \frac{c}{x}$

$$y' = -\frac{c}{x^2}$$

Multiply both sides by x.

$$xy' = -\frac{c}{x^2}(x)$$

$$xy' = -\frac{c}{x}$$

$\therefore y = \frac{c}{x}$ (put in above equ)

Hence $xy' = -y$. the given ODE.

Example #2:-

$$y' = \frac{dy}{dx} = 3y \ , \ y = ce^{3x} \ , \ y(0) = 5.7.$$

Sol:-

The general solution is $y = ce^{3x}$.

initial condition willbe

$$y(0) = ce^{3(0)}$$
$$= ce^{0} = c \Rightarrow 5.7$$

Hence, the initial value problem has the solution

$$y(x) = 5.7e^{3x}.$$

This is the particular solution.

Example #3:-

$$y' = 0.5y \quad , \quad y = ce^{0.5x} \quad , y(2) = 2.$$

Sol:-

as $y(2) = 2$ then

$$y(2) = ce^{0.5(2)} \Rightarrow ce^{(1/2 \times 2)}$$

$$y(2) = ce \qquad \because y(2) = 2$$

$$ce = 2$$

$$c = \frac{2}{e}$$

$$y = \frac{2}{e} e^{0.5x}$$

$y = 0.736\, e^{0.5x}$, is the particular solution.

$\cancel{y' = 0.5(ce^{0.5x})}$

Example #4:-

ODE $(y')^2 - xy' + y = 0. \rightarrow ①$

general solution $y = cx - c^2$ and singular solution $y = \frac{x^2}{4}$.

Sol:-

Differentiate $y = cx - c^2$

$y' = c$ put the values of y and y' in equ ①

$$(y')^2 - xy' + y = 0$$

$$\cancel{c^2} - \cancel{xc} + \cancel{cx} - \cancel{c^2} = 0$$

$$0 = 0$$

Hence, satisfied that it is the general solution.

Now differentiate. $y = \frac{x^2}{4}$

$$y' = \frac{x}{2}$$

Put y and y' in $(y')^2 - xy' + y = 0$

$$\left(\frac{x}{2}\right)^2 - x\left(\frac{x}{2}\right) + \frac{x^2}{4} = 0$$

$$\frac{x^2}{4} - \frac{x^2}{2} + \frac{x^2}{4} = 0$$

$$\cancel{0.25x^2} - \cancel{0.5x^2} + \cancel{0.25x^2} = 0$$

$0 = 0$, Hence proved.

Example #5:-

$$2y\,dy = (x^2+1)\,dx$$

Sol:-

The given equ. is already in seperable form. so we will simply integrate both sides

$$\int 2y\,dy = \int (x^2+1)\,dx$$

$$\int 2y\,dy = \int x^2\,dx + \int 1\,dx$$

$$\cancel{2}\left[\frac{y^2}{\cancel{2}}\right] = \frac{x^3}{3} + x + c$$

$$y^2 = \frac{x^3}{3} + x + c$$

Example #6:

$$\frac{dy}{dx} + \frac{1}{2}y = \frac{3}{2} \quad , y(0) = 2$$

Sol:

As the above equation is not in seperable form so we will first separate the variables.

$$\frac{dy}{dx} + \frac{1}{2}y = \frac{3}{2}$$

$$\frac{dy}{dx} = \frac{3}{2} - \frac{1}{2}y$$

$$\frac{dy}{dx} = \frac{1}{2}[3-y]$$

$$\frac{1}{3-y}\,dy = \frac{1}{2}\,dx$$

Now integrate the both sides and as $y(0)=2$ so, the interval will be $x=0$ to x and $y(0)=2$ to y so,

$$\int_2^y \frac{dy}{3-y} = \int_0^x \frac{1}{2}\,dx$$

$$-\left[\ln(3-y)\right]_2^y = \frac{1}{2}\left[x\right]_0^x$$

If we multiply both sides by '-1' then

$$\left(\ln[3-y]\right)_2^y = -\frac{1}{2}(x)_0^x$$

$$\ln[3-y] - \ln[3-2] = -\frac{1}{2}(x-0)$$

$$\ln(3-y) - \cancel{\ln(1)}^{\,0} = -\frac{1}{2}x$$

$$\ln(3-y) = -\frac{1}{2}x.$$

Taking exponential on both sides

$$e^{\ln(3-y)} = e^{-1/2x}$$
$$3-y = e^{-1/2x}$$

$$-y = e^{-1/2x} - 3$$

$$y = 3 - e^{-1/2x} \quad \underline{\text{Ans}}$$

For particular solution, $y(0) = 2$ then

$$y = 3 - e^{-1/2(0)}$$

$$y = 3 - e^{0}$$
$$y = 3 - 1 \Rightarrow 2$$

Hence proved.

Example #7:-

$$2xyy' = y^2 - x^2$$

Sol:-

$$2xyy' = y^2 - x^2 \rightarrow (1)$$

Now to separate in reducible form we will first divide equ (1) with $2xy$.

$$(\cancel{2yx}y')\frac{1}{\cancel{2xy}} = \frac{y^2-x^2}{2xy}$$

$$y' = \frac{y^2-x^2}{2xy}$$

$$y' = \frac{y^2}{2xy} - \frac{x^2}{2xy}$$

$$y' = \frac{y}{2x} - \frac{x}{2y} \rightarrow (2)$$

Now let $v = y/x$ & $\frac{x}{y} = \frac{1}{v}$

and differentiating $v = y/x$

$$y = vx$$
$$y' = v'x + v$$

so putting these in equ(2)

$$y' = \frac{y}{2x} - \frac{x}{2y}$$

$$v'x + v = \frac{v}{2} - \frac{1}{2v}$$

$$v'x = -v + \frac{v}{2} - \frac{1}{2v}$$

$$v'x = -\frac{v}{2} - \frac{1}{2v}$$

$$v'x = \frac{-v^2 - 1}{2v}$$

$$\frac{dv}{dx}x = \frac{-v^2-1}{2v}$$

$$\frac{dv}{dx}x = -\frac{(v^2+1)}{2v}$$

$$\frac{2v}{v^2+1}dv = -\frac{1}{x}dx$$

Now integrate both sides

$$\int \frac{2v}{v^2+1}\,dv = -\int \frac{1}{x}\,dx$$

$$\ln|v^2+1| = -\ln|x| + \ln|c|$$

$$\ln|v^2+1| = \ln(c/x)$$

Now taking exponents on both sides gets

$$v^2+1 = \frac{c}{x}$$

Now put $v = (y/x)$

$$y^2/x^2 + 1 = \frac{c}{x}$$

$$\frac{y^2+x^2}{x^2} = \frac{c}{x}$$

$$x^2+y^2 = \frac{c}{\cancel{x}}(x^{\cancel{2}})$$

$$x^2+y^2 = cx.$$

PROBLEM #1:-

$$y' + (x+2)y^2 = 0$$

Sol:-

$$\frac{dy}{dx} + (x+2)y^2 = 0$$

$$\frac{dy}{dx} = -(x+2)y^2$$

$$\frac{1}{y^2}\,dy = -(x+2)\,dx$$

Now integrate both sides

$$\int \frac{1}{y^2}\,dy = -\int (x+2)\,dx$$

$$\int y^{-2}\,dy = +\left[\int -x\,dx - 2\int dx\right]$$

$$\frac{-1}{y} = +\left[-\frac{x^2}{2} - 2x + c\right]$$

$$-\frac{1}{y} = -\frac{x^2}{2} - 2x + c$$

$$-\frac{1}{y} = \frac{-x^2 - 4x + 2c}{2} \qquad \therefore 2c = c$$

$$-\frac{1}{y} = -\frac{x^2 - 4x + c}{2}$$

$$y = \frac{2}{x^2 + 4x + c}$$

PROBLEM #2:

$$yy' + 4x = 0 \,, \quad y(0) = 3$$

Sol:

$$y\frac{dy}{dx} + 4x = 0$$

$$y\frac{dy}{dx} = -4x$$

$$y\,dy = -4x\,dx$$

Now integrate both sides

$$\int y\,dy = -4\int x\,dx$$

$$\frac{y^2}{2} = -\cancel{4}^{2}\left(\frac{x^2}{\cancel{2}_1}\right) + c$$

$$\frac{y^2}{2} = -2x^2 + c$$

$$y^2 = 2(-2x^2 + c)$$

$$y^2 = -4x^2 + 2c \qquad \therefore 2c = c$$

$$y^2 = -4x^2 + c. \rightarrow (1)$$

Now, $y(0) = 3$ put in equ (1)

$$y^2 = -4x^2 + c$$

$$(3)^2 = -4(0) + c$$

$$9 = \cancel{-0} + c$$

$$c = 9$$

Hence, $y^2 = -4x^2 + 9$ is the particular solution.