LECTURE #5

day / date: TUE / 26 MAR, 19

:- SOLVED EXAMPLES :-

EXAMPLE #1:- $y' + 1.25y = 5$, $y(0) = 6.6$

Sol:-

Solved in lecture #4, PROBLEM #1.

EXAMPLE #2:-

$$y' + ty = ty^3 \rightarrow (1)$$

Sol:-

Linear equ.:- $y' + p(x)y = r(x)$

Now as we see $a = 3$ then

$v = y^{1-a} = y^{1-3} \Rightarrow y^{-2}$ Now differentiate it & multiply with equ(1).

$$v' = -2y^{-2-1} \Rightarrow -2y^{-3}$$

$$y' + ty = ty^3$$

$$v' = (-2y^{-3})y' + (-2y^{-3})ty = (-2y^{-3})ty^3$$

$$v' = -2y^{-3}y' - 2y^{-3+1}t = -2y^{-3+3}t$$

$$= -2y^{-3}y' - 2y^{-2}t = -2y^{0}t$$

$$v' = -2y^{-3}y' - 2y^{-2}t = -2t$$

$$v' = -2y^{-3}y' = 2y^{-2}t - 2t$$

$$v' = 2y^{-2}t - 2t \rightarrow (2)$$

As $v = y^{-2}$ then equ (2) becomes

$$v' = 2vt - 2t$$
$$v' - 2vt = -2t$$ linear ODE, transformed.

Now $r = -2t$, $p = -2t$ then

Integrating factor $= I.F = h = \int p\,dt$

$$= \int -2t\,dt = -2\int t\,dt$$

$$h = -\cancel{2}\frac{t^2}{\cancel{2}} \Rightarrow -t^2$$

then

$$y = e^{-h}\left[\int e^{h} r\,dt + c\right]$$

$$v \cancel{y} = e^{-(-t^2)}\left[\int e^{-t^2}(-2t)\,dt + C\right]$$

let's substitute $v = t^2$, $v' = 2t\,dt$

then

$$\int e^{-t^2}(-2t)dt \Rightarrow -\int e^{-v}\,dv \Rightarrow e^{-v} \Rightarrow e^{-t^2}$$

so,

$$v = e^{t^2}\left[e^{-t^2} + C\right]$$

$$v = e^{t^2 - t^2} + e^{t^2}C \Rightarrow e^0 + e^{t^2}C$$
$$v = 1 + Ce^{t^2}$$

Now as $u = y^{-2} \Rightarrow \frac{1}{y^2}$

$$v^{-1/2} = y$$ then

$$y = v^{-1/2} = \left[1 + Ce^{t^2}\right]^{-1/2}$$ general solution.

KAGHAZ

Example #3:

$$y' + x^2 y = \frac{e^{-x^3}\sinh x}{3y^2} \rightarrow (1)$$

Sol:

Bernoulli's equ: $y' + p(x)y = g(x)y^a$

Now, $u = y^{1-a}$ $\quad \because a = -2$

$u = y^{1+2} \Rightarrow y^3$

Now differentiate it,

$$\frac{du}{dy} = 3y^2 \text{ or } u' = 3y^2 y'$$

Multiply $3y^2$ with equ(1)

$$3y^2 y' + x^2 y(3y^2) = \frac{e^{-x^3}\sinh x}{\cancel{3y^2}}(\cancel{3y^2})$$

$$3y^2 y' + 3x^2 y^3 = e^{-x^3}\sinh x \rightarrow (2)$$

Now as $u' = 3y^2 y'$ and $u = y^3$ hence equ(2) becomes

$$u' + 3x^2 u = e^{-x^3}\sinh x \rightarrow \text{linear form}$$

$p = 3x^2$, $r = e^{-x^3}\sinh x$

$h = \int p(x)dx = \int 3x^2 dx$

$$h = 3\int x^2 dx = 3\left[\frac{x^3}{3}\right] \Rightarrow x^3$$

then

$$u = e^{-x^3}\left[\int e^{x^3} e^{-x^3}\sinh x\, dx + c\right]$$

$$= e^{-x^3}\left[\int e^{\cancel{x^3} - \cancel{x^3}}\sinh x\, dx + c\right]$$

$$= e^{-x^3}\left[\int e^0 \sinh x\, dx + c\right]$$

$$u = e^{-x^3}\left[\int \sinh x\, dx + c\right]$$

$$U = e^{-x^3}\left[\cosh x + c\right]$$

$$U = e^{-x^3}\cosh x + e^{-x^3}C$$

Now $U = y^3$, $y = U^{1/3}$

$$y = \left[e^{-x^3}\cosh x + e^{-x^3}C\right]^{1/3} \Rightarrow \text{general solution}$$

EXAMPLE #4:-

$$y_1 = \cos(x), \quad y_2 = \sin(x)$$

$$y'' + y = 0$$

Sol:-

let's consider function $y = \cos(x)$

$y' = -\sin x$, $y'' = -\cos x$.

$y'' + y = 0$, $-\cos x + \cos x = 0$ Hence verified.

$0 = 0$

Now let's multiply $\cos x$ and $\sin x$ with the constants say by 2 with $\cos x$, 4 with $\sin x$ and take the sum of the result

$$y = c_1 y_1 + c_2 y_2$$

$$y = 2\cos x - 4\sin x$$

$$y' = -2\sin x - 4\cos x$$
$$y'' = -2\cos x - (-4\sin x)$$
$$y'' = -2\cos x + 4\sin x$$

Now,

$$y''+y=0$$

$$-2\cancel{\cos x}+4\cancel{\sin x}+2\cancel{\cos x}-4\cancel{\sin x}=0$$

$$0=0$$

Hence proved.

EXAMPLE #5:-

$$y''+y=0\ ,\ y(0)=3.0,\ y'(0)=-0.5$$

Sol:-

The functions $\cos x$ and $\sin x$ are the solutions of the ODE $y''+y=0$ as we considered in example(4).

so,

$$y=c_1\cos x+c_2\sin x \text{ (general solution)}$$

For particular solution:-

$$y'=-c_1\sin x+c_2\cos x$$

$$y(0)=c_1\cos(0)+c_2\sin(0)$$

$$3.0=c_1(1)+0 \qquad \because \cos(0)=1\ ,\ \sin(0)=0$$

$$\text{so } c_1=3.0.$$

$$y'(0)=-c_1\sin(0)+c_2\cos(0)$$

$$-0.5=-0+c_2$$

$$c_2=-0.5.$$

Hence the particular solution will be then,

$$y=3.0\cos x-0.5\sin x.$$