ISRA UNIVERSITY
Islamabad Campus

Program: BSc
Semester – Spring 2019

MTCA-183
Calculus-II
Solution

Quiz – 1
Marks: 10 **Handout Date: 19/03/2019**

Question # 1:
Find the general solution of the following equation using method of exact differential equations:

$$(6x^2 - y + 3)dx + (3y^2 - x - 2)dy = 0$$

Solution:

$$(6x^2 - y + 3)dx + (3y^2 - x - 2)dy = 0$$

Check for exactness:

$$M = 6x^2 - y + 3, \ \ N = 3y^2 - x - 2$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}[6x^2 - y + 3] = -1 \ , \quad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}[3y^2 - x - 2] = -1$$

Hence:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

The given differential equation is exact.

The general solution u (x, y) = c is given by:

$$u(x, y) = \int M dx + k(y)$$

$$u(x, y) = \int (6x^2 - y + 3)dx + k(y)$$

$$u(x, y) = \frac{6x^3}{3} - xy + 3x + k(y) = 2x^3 - xy + 3x + k(y)$$

Now to find k (y) lets differentiate the above equation:

$$u(x, y) = \frac{\partial}{\partial y}[2x^3 - xy + 3x + k(y)] = N(x, y)$$

$$u(x, y) = -x + k'(y) = 3y^2 - x - 2$$

$$k'(y) = 3y^2 - 2$$

Now integrate k' (y):

$$\int k'(y)\,dy = \int 3y^2 - 2\,dy$$

$$k(y) = \frac{3y^3}{3} - 2y + c_1 = y^3 - 2y + c_1$$

Now:

$$u(x,y) = 2x^3 - xy + 3x + y^3 - 2y + c_1 = c$$

The general solution is:

$$2x^3 - xy + 3x + y^3 - 2y = c$$

Good Luck