LECTURE #7

day / date: TUE / 7-5-19

SOLVED EXAMPLES

EXAMPLE #1

$$y'' - 5y' + 6y = 0$$

Sol:

The characteristic equation will be:-

$$\lambda^2 - 5\lambda + 6 = 0$$

which factors to:-

$$(\lambda - 2)(\lambda - 3) = 0$$

$$\lambda - 2 = 0 \quad , \quad \lambda - 3 = 0$$

$$\lambda = 2 \quad , \quad \lambda = 3$$

which in turn means.

$$y_1 = e^{2x} \quad , \quad y_2 = e^{3x}$$

are solutions to our original differential equation

$$y(x) = c_1 e^{2x} + c_2 e^{3x} \Rightarrow \text{general solution.}$$

EXAMPLE #2

$$y'' + y' - 2y = 0 \quad , \quad y(0) = 4 \quad , \quad y'(0) = -5$$

Sol:

The characteristic equation.

$$\lambda^2 + \lambda - 2 = 0$$

According to quadratic formula.

$$\lambda_1 = \frac{1}{2}\left(-1 + \sqrt{1+8}\right) \quad , \quad \lambda_2 = \frac{1}{2}\left(-1 - \sqrt{9}\right)$$

$$= \frac{1}{2}\left(-1 + \sqrt{9}\right) \quad , \quad = \frac{1}{2}(-1-3)$$

$$= \frac{1}{2}(-1+3) = \frac{2}{2} \Rightarrow 1, \quad = \frac{(-4)}{2} \Rightarrow -2$$

The general solution will be:-

$$y(x) = c_1 e^{x} + c_2 e^{-2x}.$$

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EXAMPLE #2

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The characteristic equation.

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The general solution will be:-

$$y(x) = c_1 e^{x} + c_2 e^{-2x}.$$

day / date:

Particular Solution:

$$y(0) = c_1 e^0 + c_2 e^0 \Rightarrow c_1 + c_2 = 4 \rightarrow (1)$$

$$y'(x) = c_1 e^x - 2c_2 e^{-2x}$$
$$y'(0) = c_1 e^0 - 2c_2 e^0 \Rightarrow c_1 - 2c_2 = -5 \rightarrow (2)$$

Subtract equ (2) from equ (1)

$$\begin{array}{rl} & c_1 + c_2 = 4 \\ \ominus & c_1 \ominus 2c_2 = \ominus 5 \\ \hline & 3c_2 = 9 \end{array}$$

$c_2 = 3$ put in equ (1)

$$c_1 + c_2 = 4$$
$$c_1 + 3 = 4$$
$$c_1 = 4 - 3 \Rightarrow 1$$

Hence, the particular solution.

$$y(x) = e^x + 3c_2 e^{-2x}.$$

EXAMPLE #3:

$$y'' + y' + 0.25y = 0 \quad , y(0) = 3, \; y'(0) = -3.5$$

Sol:

The characteristic equ is:

$$\Lambda^2 + \Lambda + 0.25 = 0$$
$$(\Lambda + 0.5)^2 = 0$$

It has double root $\Lambda = -0.5$

then the general solution is

$$y = (c_1 + c_2 x)\, e^{-0.5x}$$
$$y' = c_2 e^{-0.5x} - 0.5(c_1 + c_2 x)\, e^{-0.5x}$$

day / date:

$$y(0) = (c_1 + c_2(0))e^0$$

$$y(0) = c_1 \Rightarrow 3.0$$

$$y'(0) = c_2 e^0 - 0.5(c_1 + c_2(0))e^0$$
$$= c_2 - 0.5c_1 \quad \because c_1 = 3.0$$
$$-3.5 = c_2 - (0.5 \times 3.0)$$
$$c_2 = -2$$

Particular solution will be,
$$y = (3 - 2x)e^{-0.5x}$$

EXAMPLE #4:

$$y'' + 0.4y' + 9.04y = 0 \quad , \cancel{y(0) = 0, y'(0) = 3}$$

Sol:

The characteristic equation is.
$$\lambda^2 + 0.4\lambda + 9.04 = 0$$
$$\lambda = -\frac{1}{2}\left(a \pm \sqrt{a^2 - 4b}\right)$$

$$\lambda_{1,2} = -\frac{1}{2}(0.4) \pm \frac{\sqrt{0.16 - 36.16}}{2}$$

$$= -0.2 \pm \frac{\sqrt{-36}}{2} = -0.2 \pm \frac{i(6)}{2}$$

$\lambda = -0.2 \pm 3i$ here $w = 3$

general solution.
$$y = e^{-0.2x}(A\cos 3x + B\sin 3x)$$

day / date:

EXAMPLE #5:-

$$20y'' + 4y' + y = 0, \quad y(0) = 3.2, \quad y'(0) = 0$$

Sol:-

The characteristic eqn is.

$$20\lambda^2 + 4\lambda + 1 = 0$$

Divide 20 on both sides

$$\lambda^2 + 0.2\lambda + 0.05 = 0$$

$$\lambda = \frac{1}{2}\left(-0.2 \pm \sqrt{-0.16}\right)$$

$$\lambda_1 = \frac{1}{2}(-0.2 + i0.4) \Rightarrow -0.1 + i0.2$$

$$\lambda_2 = -0.1 - i0.2$$

$$y = e^{-0.1x}(A\cos 0.2x + B\sin 0.2x)$$

$$y' = -0.1Ae^{-0.1x}\cos 0.2x + A\,0.2e^{-0.1x}(-\sin 0.2x) - B0.1e^{-0.1x}\sin 0.2x + 0.2e^{-0.1x}B\cos 0.2x$$

$$y(0) = e^0(A\cos(0) + B\sin(0))$$

$$A = 3.2$$

$$y'(0) = -Ae^0 0.1\cos(0) + A0.2e^0(\sin(0)) - B0.1e^0\sin(0) + 0.2e^0 B\cos(0)$$

$$= -A0.1 + B0.2$$

$$0 = -A0.1 + B0.2 \qquad \because A = 3.2$$

$$0 = -(3.2)(0.1) + B0.2$$

$$0.2B = 0.32$$

$$B = 1.6$$

$$y(x) = e^{-0.2x}(3.2\cos 0.2x + 1.6\sin 0.2x)$$

day / date:

EXAMPLE #6 or

$$y'' - 6y' - 7y = 0$$

Sol:

The characteristic equation:

$$r^2 - 6r - 7 = 0$$

$$r_1 = \frac{1}{2}\left(-a + \sqrt{a^2 - 4b}\right), \quad r_2 = \frac{1}{2}\left(-a - \sqrt{a^2 - 4b}\right)$$

$$= \frac{1}{2}\left(+6 + \sqrt{36 + 28}\right), \quad = \frac{1}{2}\left(6 - \sqrt{36 + 28}\right)$$

$$r_1 = \frac{1}{2}(6 + 8) \Rightarrow 7, \quad r_2 = \frac{1}{2}(6 - 8) \Rightarrow -1.$$

$y = c_1 e^{7x} + c_2 e^{-x}$ general solution.