day / date: TUE/14 MAY,19

LECTURE #8

& SOLVED EXAMPLES:-

EXAMPLE #1:-

$P(D) = D^2 - 3D - 4I$ and solve for $P(D)y = 0$.

SOL:-

as $I^2 = I$ then

$$D^2 - 3D - 4I = (D - 4I)(D + I)$$

Now

$$(D - 4I)y \Rightarrow y' - 4y = 0$$
$$\lambda - 4 = 0$$
$$\lambda = 4$$
$$y_1 = e^{\lambda_1 x} = e^{4x}$$
$$(D + I)y \Rightarrow y' + y = 0$$
$$\lambda + 1 = 0$$
$$\lambda = -1$$
$$y_2 = e^{\lambda_2 x} = e^{-x}$$
$$(D - 4I)(D + I)y \Rightarrow (D - 4I)(y' + y)$$
$$= D(y' + y) - 4(y' + y)$$
$$\Rightarrow y'' + y' - 4y' - 4y$$
$$\Rightarrow y'' - 3y' - 4y = 0.$$

EXAMPLE #2:-

$$(D^2 + 4.00D + 3.36I)y = 0$$

SOL:-

$$D^2 + 4.00D + 3.36I = 0 \text{ or } y'' + 4.00y' + 3.36y = 0$$

$$y'' + 4.00y' + 3.36y = 0$$
$$\lambda^2 + 4.00\lambda + 3.36 = 0$$

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$$r = \frac{-4.00 \pm \sqrt{(4.00)^2 - 4(3.36)}}{2}$$

$$= \frac{-4.00 \pm \sqrt{16 - 13.44}}{2}$$

$$r = \frac{-4.00 \pm 1.6}{2}$$

$$r_1 = \frac{-4.00 + 1.6}{2} \quad , \quad r_2 = \frac{-4.00 - 1.6}{2}$$

$$r_1 = -1.2 \quad , r_2 = -2.8$$

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

$$y = c_1 e^{-1.2x} + c_2 e^{-2.8x}.$$ general solution.

EXAMPLE #3:-

$$x^2 y'' - 5xy' + 9y = 0$$

Sol:-

$x^2 y'' - 5xy' + 9y = 0$ Euler - Cauchy Equation

$$y = x^m \quad , y' = m x^{m-1} \quad , y'' = m(m-1) x^{m-2}$$

$$x^2 [m(m-1)] x^{m-2} - 5x (m x^{m-1}) + 9 x^m = 0$$

$$x^{m-2+2} [m^2 - m] - 5m x^{m-1+1} + 9 x^m = 0$$

$$x^m [m^2 - m] - 5m x^m + 9 x^m = 0$$

$$x^m [m^2 - m - 5m + 9] = 0$$

$$m^2 - 6m + 9 = 0. \text{ Auxiliary eqn.}$$

$$(m-3)(m-3) = 0$$

$$m = 3$$

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$$y = (c_1 + c_2 \ln x)\, x^m$$

$$y = (c_1 + c_2 \ln x)\, x^3.$$

EXAMPLE #4:

$$x^2 y'' + 0.6xy' + 16.04\, y = 0$$

Sol:-

$$y = x^m \;,\; y' = mx^{m-1} \;,\; y'' = m(m-1)x^{m-2}$$

$$x^2\left[m^2 - m\right]x^{m-2} + 0.6x(mx^{m-1}) + 16.04\,x^m = 0$$

$$x^m\left[m^2 - m\right] + 0.6mx^m + 16.04\,x^m = 0$$

$$x^m\left[m^2 - m + 0.6m + 16.04\right] = 0$$

$$m^2 - 0.4m + 16.04 = 0 \quad \text{Auxiliary equ.}$$

$$m = \frac{+0.4 \pm \sqrt{(0.4)^2 - 4(16.04)}}{2}$$

$$m = \frac{+0.4 \pm \sqrt{0.16 - 64.16}}{2}$$

$$m = \frac{0.4 \pm \sqrt{-64}}{2}$$

$$m_1 = \frac{0.4 + 8i}{2} \;,\; m_2 = \frac{0.4 - 8i}{2}$$

$$m_1 = 0.2 + 4i \;,\; m_2 = 0.2 - 4i$$

Now let $x = e^{\ln x}$

$$x^{m_1} = x^{0.2+4i} = x^{0.2}(e^{\ln x})^{4i} \Rightarrow x^{0.2}\, e^{4\ln x i}$$

$$x^{m_2} = x^{0.2-4i} = x^{0.2}(e^{\ln x})^{-4i} \Rightarrow x^{0.2}\, e^{-4i\ln x}$$

$\because\; e^{it} = \cos t + i\sin t$

$t = 4\ln x$

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$$x^{m_1} = x^{0.2}\left[\cos(4\ln x) + i\sin(4\ln x)\right]$$
$$x^{m_2} = x^{0.2}\left[\cos(4\ln x) - i\sin(4\ln x)\right]$$

Add these two formulas, so that sin drops out and divide the result by 2.

$$x^{m_1} = x^{0.2}\cos(4\ln x) + x^{0.2} i\sin(4\ln x)$$
$$+\ x^{m_2} = x^{0.2}\cos(4\ln x) - x^{0.2} i\sin(4\ln x)$$

$$= \frac{2x^{0.2}\cos(4\ln x)}{2} \Rightarrow x^{0.2}\cos(4\ln x)$$

Then subtract and divide by $2i$

$$x^{m_1} = x^{0.2}\cos(4\ln x) + x^{0.2} i\sin(4\ln x)$$
$$-x^{m_2} = \ominus x^{0.2}\cos(4\ln x) \pm x^{0.2} i\sin(4\ln x)$$

$$= \frac{2ix^{0.2}\sin(4\ln x)}{2i}$$
$$\Rightarrow x^{0.2}\sin(4\ln x)$$

general solution:

$$y = x^{0.2}\left[A\cos(4\ln x) + B\sin(4\ln x)\right].$$

Example #5:

$$x^2y'' - 6y = 0$$

Sol:

$x^2y'' + axy' + by = 0$ standard form.

$$m^2 - 6 = 0$$
$$m = \frac{-0 \pm \sqrt{0^2 - 4(-6)}}{2}$$
$$m = \frac{\pm\sqrt{24}}{2}$$

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$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$x^2y'' + 0xy' - 6y = 0$

$m(m-1)x^{m-2+2} + mx^{m-1+1}(0) - 6x^m = 0$

$(m^2 - m)x^m + 0 - 6x^m = 0$

$x^m[m^2 - m - 6] = 0$

$m^2 - m - 6 = 0$ Auxiliary eqn.

$$m = \frac{+1 \pm \sqrt{(-1)^2 - 4(-6)}}{2}$$

$$= \frac{1 \pm \sqrt{1+24}}{2}$$

$$m_1 = \frac{1+\sqrt{25}}{2} \quad , m_2 = \frac{1-\sqrt{25}}{2}$$

$$m_1 = \frac{1+5}{2} \quad , m_2 = \frac{1-5}{2}$$

$$m_1 = \frac{6}{2} \Rightarrow 3 \quad , m_2 = \frac{-4}{2} = -2$$

$y = c_1x^3 + c_2x^{-2}$ general solution.

EXAMPLE#6:

$x^2y'' - 4xy' + 6y = 0$, $y(1) = 1$, $y'(1) = 0$

Sol:

$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$(x^{m-2+2})(m^2 - m) - 4x^{m-1+1}m + 6x^m = 0$

$x^m[m^2 - m - 4m + 6] = 0$

$m^2 - 5m + 6 = 0$ Auxiliary eqn.

day / date:

$$m^2 - 5m + 6 = 0$$

$$m = \frac{+5 \pm \sqrt{(-5)^2 - 4(6)}}{2}$$

$$m = \frac{5 \pm \sqrt{25-24}}{2}$$

$$m_1 = \frac{5+1}{2} \quad , \quad m_2 = \frac{5-1}{2}$$

$$m_1 = \frac{6^3}{2} \Rightarrow 3 \quad , \quad m_2 = \frac{4^2}{2} \Rightarrow 2$$

$y = c_1 x^3 + c_2 x^2$ general solution

$y(1) = 1$

$$y(1) = c_1 (1)^3 + c_2 (1)^2$$

$$1 = c_1 + c_2 \rightarrow (1)$$

$$y' = 3c_1 x^2 + 2c_2 x$$

$$y'(1) = 3c_1 (1)^2 + 2c_2 (1)$$

$$3c_1 + 2c_2 = 0 \rightarrow (2)$$

Multiply equ (1) by 3 and subtract with equ (2)

$$3c_1 + 3c_2 = 3$$

$$\ominus 3c_1 \ominus 2c_2 = \ominus 0$$

$$\boxed{c_2 = 3}$$ put in equ (1)

$$c_1 + c_2 = 1$$

$$c_1 + 3 = 1$$

$$c_1 = 1 - 3 \Rightarrow -2.$$

day / date:

$y = -2x^3 + 3x^2$ particular solution