Program: BSc
Semester – Spring 2019

MTCA-183
Calculus-II

Quiz – 1
Marks: 10   **Handout Date: 07/05/2019**

Question # 1:
Verify by substitution that $y_1 = e^{4x}$ and $y_2 = e^{2x}$ are solution of the ODE $y'' - 6y' + 8y = 0$, solve for the initial value problem $y(0) = 3$ and $y'(0) = 2$.

Solution:
Let's check:

$$y_1 = e^{4x}$$
$$y_1' = 4e^{4x}, y_1'' = 16e^{4x}$$
$$y'' - 6y' + 8y = 0$$
$$16e^{4x} - 6(4e^{4x}) + 8e^{4x} = 0$$
$$16e^{4x} - 24e^{4x} + 8e^{4x} = 0$$
$$0 = 0, hence\ proved.$$

Now let's check:

$$y_2 = e^{2x}$$
$$y_2' = 2e^{2x}, y_2'' = 4e^{2x}$$
$$y'' - 6y' + 8y = 0$$
$$4e^{2x} - 6(2e^{2x}) + 8e^{2x} = 0$$
$$4e^{2x} - 12e^{2x} + 8e^{2x} = 0$$
$$0 = 0, hence\ proved.$$

As y_1 and y_2 are the solutions of the given ODE the general solution is:
$$y(x) = c_1 y_1 + c_2 y_2$$
$$y(x) = c_1 e^{4x} + c_2 e^{2x}$$

Now for particular solution:
$$y'(x) = 4c_1 e^{4x} + 2c_2 e^{2x}$$
$$y(0) = c_1 e^0 + c_2 e^0 \Rightarrow 3 = c_1 + c_2 \rightarrow (1)$$
$$y'(0) = 4c_1 e^0 + 2c_2 e^0 \Rightarrow 2 = 4c_1 + 2c_2 \rightarrow (2)$$

Now multiply eq (1) by 2 and add with eq (2):
$$2c_1 + 2c_2 = 6$$
$$\mp 4c_1 \mp 2c_2 = \mp 2$$
$$-2c_1 = 4$$
$$c_1 = -2, put\ in\ equ\ (1)$$

$$c_1 + c_2 = 3 \Rightarrow -2 + c_2 = 3$$
$$c_2 = 5$$

Hence:

$$y(x) = -2e^{4x} + 5e^{2x}$$