Quiz – 3a
Marks: 20   **Handout Date: 14/05/2019**

Question # 1:
Verify by substitution that $y_1 = e^x$ and $y_1 = e^{-x}$ are solution of the ODE $y'' - y = 0$, solve for the initial value problem $y(0) = 6$ and $y'(0) = -2$.

Solution:
Let's check:

$$y_1 = e^x$$
$$y_1' = e^x, y_1'' = e^x$$
$$y'' - y = 0$$
$$e^x - e^x = 0$$
$$0 = 0, hence\ proved.$$

Now let's check:

$$y_2 = e^{-x}$$
$$y_2' = -e^{-x}, y_2'' = e^{-x}$$
$$y'' - y = 0$$
$$e^{-x} - e^{-x} = 0$$
$$0 = 0, hence\ proved.$$

As y_1 and y_2 are the solutions of the given ODE the general solution is:
$$y(x) = c_1 y_1 + c_2 y_2$$
$$y(x) = c_1 e^x + c_2 e^{-x}$$

Now for particular solution:
$$y'(x) = c_1 e^x - c_2 e^{-x}$$
$$y(0) = c_1 e^0 + c_2 e^{-0} \Rightarrow 6 = c_1 + c_2 \rightarrow (1)$$
$$y'(0) = c_1 e^0 - c_2 e^{-0} \Rightarrow -2 = c_1 - c_2 \rightarrow (2)$$

Now add eq (1) and eq (2):
$$c_1 + c_2 = 6$$
$$\mp c_1 \pm c_2 = \pm 2$$
$$2c_2 = 8$$
$$c_2 = 4, put\ in\ equ\ (1)$$
$$c_1 + c_2 = 6 \Rightarrow c_1 + 4 = 6$$
$$c_1 = 2$$

Hence:

$$y(x) = 2e^x + 4e^{-x}$$

Question # 2:
Solve the IVP. Show the details of your work.
$$y'' + y' - 6y = 0, y(0) = 10, y'(0) = 0$$

Solution:
The characteristic equation will be:
$$\lambda^2 + \lambda - 6 = 0$$

Using Quadratic equation:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-1 \pm \sqrt{1^2 - 4(-6)}}{2}$$

$$\lambda = \frac{-1 \pm \sqrt{1 + 24}}{2} = \frac{-1 \pm 5}{2}$$

$$\lambda_1 = \frac{-1 + 5}{2} = \frac{4}{2} \Rightarrow 2, \qquad \lambda_2 = \frac{-1 - 5}{2} = -\frac{6}{2} \Rightarrow -3$$

Then the general solution is:
$$y(x) = c_1 e^{2x} + c_2 e^{-3x}$$

Now for particular solution:
$$y'^{(x)} = 2c_1 e^{2x} - 3c_2 e^{-3x}$$
$$y(0) = c_1 e^0 + c_2 e^0 \Rightarrow 10 = c_1 + c_2 \rightarrow (1)$$
$$y'(0) = 2c_1 e^0 - 3c_2 e^0 \Rightarrow 0 = 2c_1 - 3c_2 \rightarrow (2)$$

Now multiply eq (1) by 2 and add with eq (2):
$$2c_1 + 2c_2 = 20$$
$$\mp 2c_1 \pm 3c_2 = \mp 0$$
$$5c_2 = 20$$
$$c_2 = 4, put\ in\ equ\ (1)$$
$$c_1 + c_2 = 10 \Rightarrow c_1 + 4 = 3$$
$$c_1 = -1$$

Hence:
$$y(x) = -1e^{2x} + 4e^{-3x}$$