



ISRA UNIVERSITY

Islamabad Campus

**Program: BSc
Semester – Spring 2019
Solution
MTCA-183
Calculus-II**

Quiz – 3b

Marks: 10

Handout Date: 14/05/2019

Question # 1:

Solve the IVP. Show the details of your work.

$$y'' + y' - 6y = 0, y(0) = 10, y'(0) = 0$$

Solution:

The characteristic equation will be:

$$\lambda^2 + \lambda - 6 = 0$$

Using Quadratic equation:

$$\begin{aligned}\lambda &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ \lambda &= \frac{-1 \pm \sqrt{1^2 - 4(-6)}}{2} \\ \lambda &= \frac{-1 \pm \sqrt{1 + 24}}{2} = \frac{-1 \pm 5}{2} \\ \lambda_1 &= \frac{-1 + 5}{2} = \frac{4}{2} \Rightarrow 2, \quad \lambda_2 = \frac{-1 - 5}{2} = -\frac{6}{2} \Rightarrow -3\end{aligned}$$

Then the general solution is:

$$y(x) = c_1 e^{2x} + c_2 e^{-3x}$$

Now for particular solution:

$$\begin{aligned}y'(x) &= 2c_1 e^{2x} - 3c_2 e^{-3x} \\ y(0) &= c_1 e^0 + c_2 e^0 \Rightarrow 10 = c_1 + c_2 \rightarrow (1) \\ y'(0) &= 2c_1 e^0 - 3c_2 e^0 \Rightarrow 0 = 2c_1 - 3c_2 \rightarrow (2)\end{aligned}$$

Now multiply eq (1) by 2 and add with eq (2):

$$\begin{aligned}2c_1 + 2c_2 &= 20 \\ \mp 2c_1 \pm 3c_2 &= \mp 0 \\ 5c_2 &= 20 \\ c_2 &= 4, \text{ put in equ (1)} \\ c_1 + c_2 &= 10 \Rightarrow c_1 + 4 = 3 \\ c_1 &= -1\end{aligned}$$

Hence:

$$y(x) = -1e^{2x} + 4e^{-3x}$$

Good Luck